QABH - Lec 24 - 30 Nov (Large AdS Black Holes)

So far, we have discussed the information paradox in flat space. We now move on to large AdS black holes. Surprisingly, these paradoxes do not rely on details of AdS/CFT, and we will supply the details that we need explicitly.

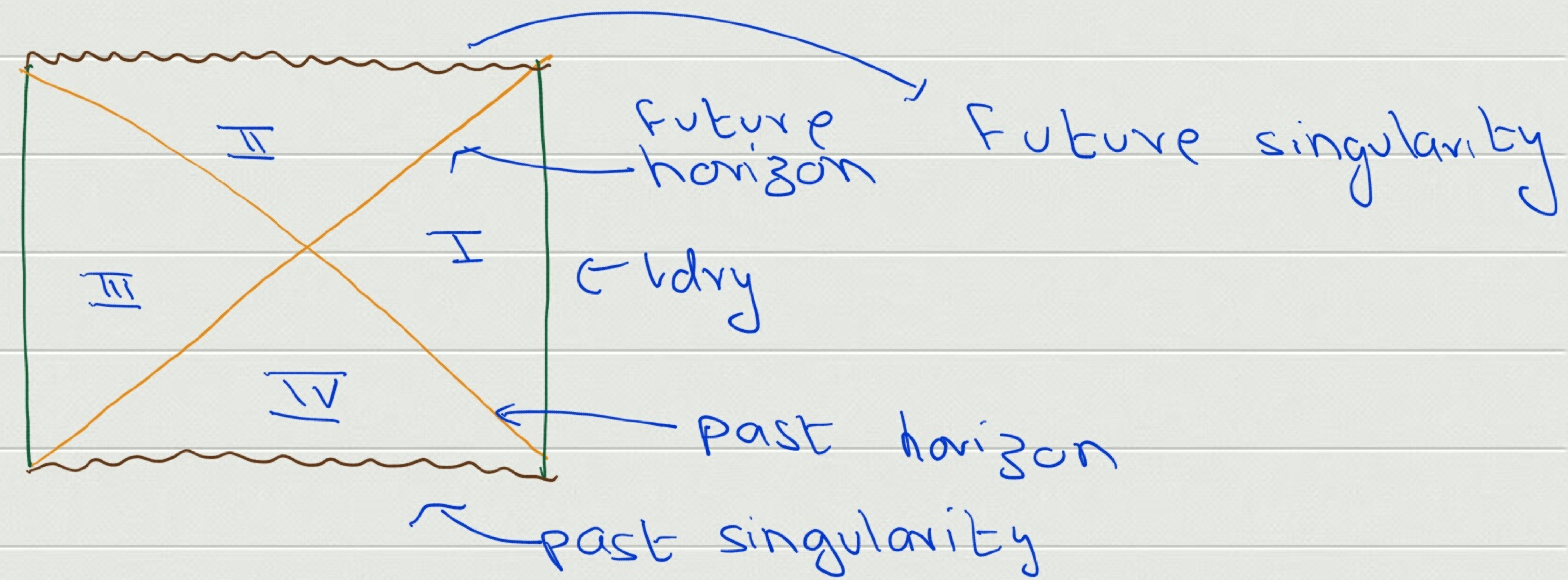
The metric of a large black hole in AdS_{d+1} is

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{d-1}$$

$$f(r) = r^2 + 1 - c_d\, G M r^{2-d} \; ; \quad c_d = 8\,\Gamma(d/2)\,\pi^{(2-d)/2}/(d-1)$$

in essence, this is very similar to the Schwarzschild solution.

The horizon is at $f(r)=0$, and the boundary at $r=\infty$. The Penrose diagram (with the maximal extension is)



A black hole formed from collapse does not have region III & IV. but in region I & II, it is similar to the eternal black hole, so, this Penrose diagram is sufficient for our purpose.

Remember that we have $\left(\frac{l_{ads}}{l_{pl}}\right)^{d-2} = N^2$

In our units, with $l_{ads}=1$

$$G = \frac{1}{N^2}$$

So for the black hole to have an $O(1)$ horizon we need

$$M \propto N^2$$

We will take

$$M = k N^2$$

where $k \gg 1$, but $k = O(1)$.

It is easy to check that $S \propto N^2$ as well.

It turns out that all extensive thermodynamic quantities have the form

$$N^2 F(E/N^2)$$

where F is some smooth function. [This is obvious from the geometry.]

Note also that $T = O(1)$ does not scale with N^2.

Also, note that

$$\frac{\partial S}{\partial E} > 0$$

Finally, from Euclidean action computations we can check that for $E \gg N^2$, the black hole saddle point dominates the thermodynamics.

The statement of AdS/CFT is just the claim that

Q. G. in AdS is exactly dual to an ordinary QFT on the boundary.

We now use this to transfer some standard results to this case

Recall our proof that generic states are exponentially close to the microcanonical ensemble.

given a state $|\psi\rangle = \sum a_i |E_i\rangle$
$E_i \in (E_0 - \Delta E, E_0 + \delta E)$

$$\langle\langle \psi | A_\alpha | \psi \rangle\rangle = \frac{1}{D_E} \sum_i \langle E_i | A_\alpha | E_i \rangle + O(e^{-S/2} \sigma_A).$$

dim of space
of states $(E_0 - \Delta E, E_0 + \Delta E)$

So a generic high energy state has the features of a black hole.

Moreover, using the equivalence of ensembles

$$\langle \psi | A_\alpha | \psi \rangle = \mathrm{Tr}\left(e^{-\beta H} A_\alpha\right) \frac{1}{Z(\beta)} + O\left(\frac{1}{N}\right)$$

The information paradox is related to the following question

Does a generic state in AdS/CFT have an empty interior and a smooth horizon?

Note this is a question about generic states.

For example black holes formed from collapse do not represent generic states, unless we manipulate them for a long time.

Does the interior exist in a generic state

$\longleftrightarrow$ are there operators in the theory, $\phi(x)$ so that $\langle \Psi | \phi_{CFT}(x_1) \ldots \phi_{CFT}(x_n) | \Psi \rangle$ are the same as

$$\langle \phi(x_1) \ldots \phi(x_n) \rangle_{\substack{\text{AdS} \\ \text{Schwarzschild B.H.}}}$$

$\uparrow$

where the red correlators are the ones obtained by quantizing quantum fields about an AdS-Schwarzschild black hole.

Using our previous techniques we can deduce the following properties about $\phi(x)$. Near the horizon in Kruskal coordinates (where $U=0$ is the horizon) we have

$$\phi \xrightarrow[U \to 0^-]{\text{outside}} \sum_l \int \frac{a_\omega}{\sqrt{\omega}} \left(e^{i\delta\omega} U^{i\beta\omega/2\pi} + V^{-i\beta\omega/2\pi} e^{-i\delta\omega} \right) Y_l(\Omega)\, d\omega$$

This is very similar to the expansion we wrote in flat space but note that here the left and right movers are related by the boundary condition at $r \to \infty$.

just inside the horizon we have

$$\phi \xrightarrow[V\to 0^+]{\text{inside}} \sum_\ell \int \frac{d\omega}{\sqrt{\omega}}\, Y_\ell(\Omega)\, e^{-iS_{\omega,\ell}} \left(\tilde{a}_\omega\, U^{-i\beta\omega/2\pi} + a_\omega\, V^{-i\beta\omega/2\pi} \right)$$

Here as in the flat space case, the left movers (proportional to V) cross smoothly but we need new modes for the right movers.

Recall this near-horizon expansion is independent of the details of the interaction terms.

Second, recall (see lectures 16 & 17) that using the short-distance behaviour of correlators, and the canonical commutation relations we require

$$\langle a_\omega a^\dagger_{\omega'} \rangle = (1-e^{-\beta\omega})^{-1}\,\delta(\omega-\omega')$$ ← two-pt fn outside the horizon

$$[a_\omega, a^\dagger_{\omega'}] = \delta(\omega-\omega')$$ ← c.c relations outside

$$[H, a_\omega] = -\omega a_\omega$$ ← negative energy

$$\langle \tilde{a}_\omega \tilde{a}^\dagger_{\omega'} \rangle = (1-e^{-\beta\omega})^{-1}\,\delta(\omega-\omega')$$ ← two pt. fn inside the horizon

$$[\tilde{a}_\omega, \tilde{a}_{\omega'}] = \delta(\omega-\omega')$$ ← c.c. inside the horizon

$$[H, \tilde{a}_\omega] = \omega \tilde{a}_\omega$$ ←

modes inside multiply $e^{-i\omega(r_* - t)}$

note - sign.

Also we have

$$\langle a_\omega \tilde{a}_{\omega'} \rangle = \langle a_\omega^+ \tilde{a}_\omega^+ \rangle = \frac{e^{-\beta\omega/2}}{1-e^{-\beta\omega/2}}$$

two pt fn across the horizon

$$[a_\omega, \tilde{a}_{\omega'}] = [a_\omega, \tilde{a}_{\omega'}^+] = 0$$

c.c. relations across the horizon and inside the horizon.

I will now describe some arguments due to AMPSS [arXiv:1304.6483] and also PR [1502.06692] that the $\tilde{a}_\omega$ do not exist.

Next time, we will find some loopholes in these arguments and describe a construction of the $\tilde{a}_\omega$.

For simplicity we define $\tilde{A}_{\omega_0} = \int f(\omega)\, \tilde{a}_\omega$ where $f(\omega)$ is sharply peaked about ω_0 and

$$[\tilde{A}_\omega, \tilde{A}_\omega^\dagger] = 1$$

Lack of a left inverse argument

We have

$$\tilde{A}_w \tilde{A}_w^{+} - \tilde{A}_w^{+} \tilde{A}_w = \underline{1}$$

or

$$\tilde{A}_w \tilde{A}_w^{+} = 1 + \tilde{A}_w^{+} \tilde{A}_w$$

$\tilde{A}_w^{+} \tilde{A}_w + 1$ is a positive operator and can be inverted. So

$$\boxed{\left(1 + \tilde{A}_w^{+} \tilde{A}_w\right)^{-1} \tilde{A}_w} \; \tilde{A}_w^{+} = 1$$

But this means that $\tilde{A}_w^+$ has a left-inverse if it exists.

But $\tilde{A}_w^+ : H_E \longrightarrow H_{E-w}$

$\dim(H_E) > \dim(H_{E-w})$

so it must be a many-to-one mapping

so it cannot have a left-inverse! so it cannot exist!

The Occupancy Argument

For the horizon to be smooth, we need

$$\langle \psi | \tilde{A}_\omega A_\omega^\dagger | \psi \rangle = \frac{1}{1 - e^{-\beta \omega}}$$

By our previous arguments (dropping $1/N$ terms)

$$Z \langle \psi | \tilde{A}_\omega A_\omega^\dagger | \psi \rangle = \mathrm{Tr}(e^{-\beta H} \tilde{A}_\omega \tilde{A}_\omega^\dagger)$$

$$= \mathrm{Tr}(\tilde{A}_\omega^\dagger e^{-\beta H} \tilde{A}_\omega)$$ $\leftarrow$ cyclicity of trace

$$= e^{-\beta w} \, \mathrm{Tr}\left(e^{-\beta H} \tilde{A}_w^{+} \tilde{A}_w\right) \leftarrow [H, \tilde{A}_w^{+}] \text{ commutator}$$

$$= e^{-\beta w} \, \mathrm{Tr}\left(e^{-\beta H}\left(\tilde{A}_w \tilde{A}_w^{+} - 1\right)\right) \leftarrow [\tilde{A}, \tilde{A}^{+}] \text{ commutator}$$

$$= Z \, e^{-\beta w} \langle \psi | \tilde{A}_w \tilde{A}_w^{+} | \psi \rangle - Z e^{-\beta w}$$

so

$$\langle \psi | \tilde{A}_w \tilde{A}_w^{+} | \psi \rangle = \frac{-e^{-\beta w}}{1 - e^{-\beta w}}$$

which is wrong & absurd since $\tilde{A}_w \tilde{A}_w^{+}$ is a +ve operator.

The $N_a \neq 0$ argument [Marolf, Polchinski arXiv:1307.4706]

With $\alpha_\omega = A_\omega - e^{-\beta\omega/2} \tilde{A}_\omega^+$

$$\beta_\omega = \tilde{A}_\omega - e^{-\beta\omega/2} A_\omega^+$$

we can define a number operator for the infalling observer

$$N_a = \alpha_\omega^+ \alpha_\omega + \beta_\omega^+ \beta_\omega .$$

We also define

$$N_b = A_\omega^+ A_\omega$$

Now the $N_a = 0$ state is not an N_ν eigenstate.
Conversely any N_ν eigenstate has
$$\langle N_\nu | N_a | N_\nu \rangle = O(1).$$

But $$[N_\nu, H] = O\left(\frac{1}{N}\right)$$

and so if we consider the microcanonical trace, we find

$$\frac{1}{D_E}\,\mathrm{tr}_E\,(N_a) = \frac{1}{D_E}\sum_E \langle E | N_a | E\rangle$$

$\uparrow$
dimension

change of basis
since $[N_i, H] \simeq 0$

$$\approx \frac{1}{D_E}\sum_{N_i} \langle N_i | N_a | N_i \rangle$$

$$= O(1)$$

$\leftarrow$ since N_a is a +ve operator
and $\langle N_i | N_a | N_i \rangle = O(1)$;

which suggests the infalling observer does not see an empty horizon!