

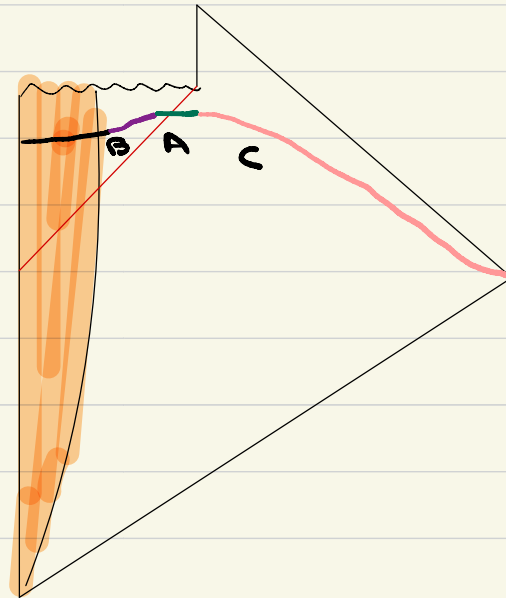
11 Feb 2020

Lecture 10: Fuzzballs, Firewalls and Holography of Information.

Comparison with original argument

The original argument by Mathur / AMPS was slightly different and based on the strong subadditivity of von Neumann entropy

Consider again the setup of the l.h.



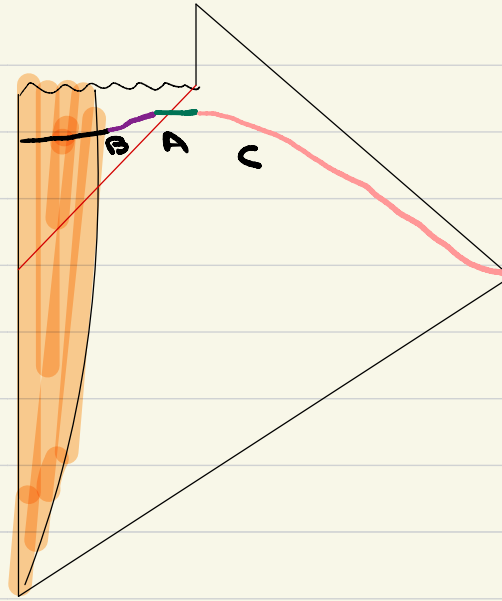
The idea is to consider the entanglement entropy of the three regions

As we emphasized, this quantity is UV-divergent. In ordinary QFT, we know how to regulate the entropy.

But in a theory of gravity, except in some special cases [such as an entanglement wedge in AdS], there is no good known definition of the E.E.

Even in AdS, none of the regions A, B, C constitute an entanglement wedge.
[In particular, C "surrounds" B and A]

So to frame the paradox, we must assume that our naive intuition will continue to hold.



First consider A and B.

The radiation in A is entangled with B.

This means that the entropy of A, B together is less than the entropy of A or of B.

$$S_{AB} < S_A; S_{AB} < S_B$$

If A, B are maximally entangled, one could

also state that

$$S_{AB} \approx 0$$

and

$$S_A \approx S_B$$

But this is not necessary to
frame the paradox.

Next, we turn to S_C . The physical picture
is that radiation is "streaming" into
C.

From our result, and under assumption of factorization
we expect S_c to follow a Page curve



Once again we have disregarded UV issues

Now at time $t + \delta t$, C gets additional radiation from A.

So the fact that

$$S_c(t + \delta t) < S_c(t) \quad [\text{for } t > t_{\text{page}}]$$

is interpreted as

$$S_{Ac} < S_c$$

But the von Neumann entropy obeys "strong subadditivity"

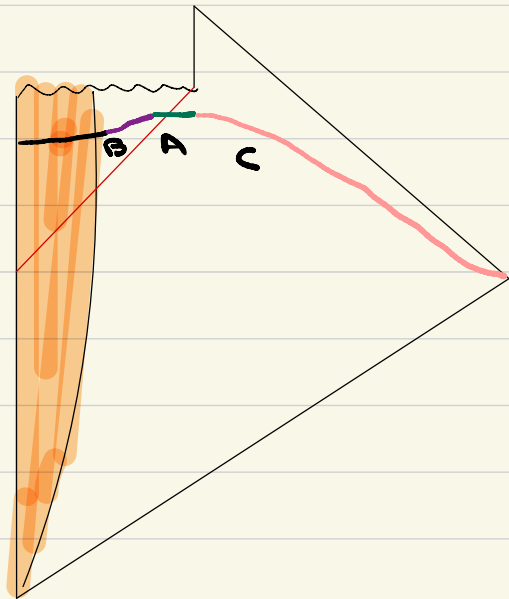
$$S_{AB} + S_{AC} \geq S_B + S_C$$

We can think of this as representing the "monogamy of entanglement"

If A, B are entangled so that S_{AB} is small

then A, C cannot be entangled and S_{AC} cannot be too small.

But here, we clearly have a contradiction



We found

$$S_{AB} < S_B$$

$$S_{Ac} < S_C$$

and so

$$S_{AB} + S_{Ac} < S_B + S_C$$

which contradicts strong subadditivity

Again the contradiction is (K1).

Historical Note.

This was formulated by Mathur in arxiv: 0909.1038

The original idea was to use "bit models", which is why one sees " $\ln 2$ " below.

Thus the entanglement entropy of the emitted quanta necessarily *increases* with each emission if the departures from the leading order Hawking state are small.

Proof: We use the strong **subadditivity** theorem [8] which governs the entropies of three systems

$$S(A+B) + S(B+C) \geq S(A) + S(C) \quad (56)$$

We set $A = \{b\}$, $B = b_{n+1}$, $C = c_{n+1}$. This gives

$$S(\{b\} + b_{n+1}) + S(p) \geq S(\{b\}) + S(c_{n+1}) \quad (57)$$

We have from (47) that $S(p) < \epsilon$. From (38) we have $S(\{b\}) = S_0$. From (50) we have $S(c_{n+1}) > \ln 2 - \epsilon$. This gives

$$S(\{b\} + b_{n+1}) > S_0 + \ln 2 - 2\epsilon \quad \square \quad (58)$$

This is the result that we wished to establish. It proves that whenever we have 'normal physics' at the horizon (i.e. the parameter giving departures from the leading order state is $\epsilon \ll 1$) then the entanglement always *increases* by at least $\ln 2 - 2\epsilon$ after each stage of evolution. The entanglement thus cannot go down at any of these stages. Thus small corrections do not change the conclusion of Hawking's leading order result, and we will get mixed states/remnants when the traditional black hole evaporates, unless we can show that the corrections to evolution are really order unity and not order $\epsilon \ll 1$.

We will consider 3 proposals

[Fill in gradually during the lecture]

Proposal	Evidence	Criticism
Fuzzballs	Supergravity sol ^{ns} Monogamy paradox	Violates EFT solutions cannot be generic
Firewalls	Monogamy paradox AMPSS paradoxes for large l.h.	Violates EFT
Holography of Information	see next few lectures!	seems to require nonlocality over long distances.

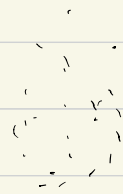
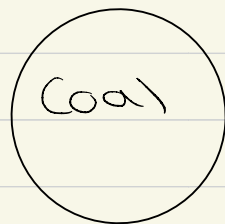
The Fuzzball proposal

The basic idea of the Fuzzball proposal is that black hole evaporation is just like the evaporation of ordinary objects.

The main difference is that

"ordinary objects have no interior."

If we consider a piece of coal, it may emit thermal radiation



↑ Thermal radiation

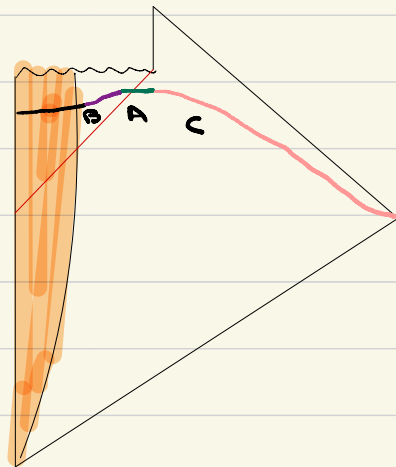
But it is **NOT TRUE** that each emitted particle has a "partner" that remains behind.

More mathematically,

we drop the idea that

$$S_{AB} < S_B ; S_{AB} < S_A$$

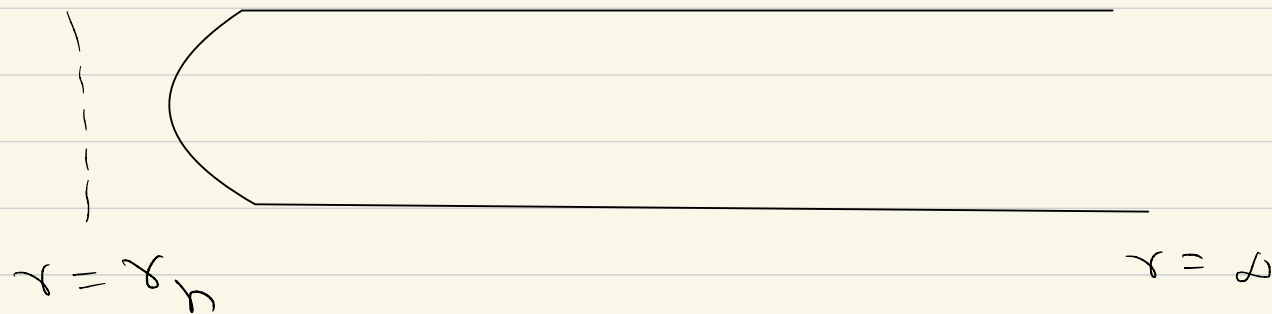
because "B" does not exist!



The fuzzball proposal suggest that perhaps space ends before the horizon is reached.

The proposal hopes to achieve this by thinking of solutions with an **extra dimension**

The idea is that one of the compact dimensions can have a profile as follows



The idea of using the extra-dimension is a clever way to sidestep the no hair theorem and Chandrasekhar limit.

There have always been proposals around to replace black holes with exotic compact objects.

The innovation of the Fuzzball proposal is to use extra-dimensions and create a topological obstruction to collapse.

A large number of such regular supersymmetric solutions have been found

Fuzzball complementarity suggests that for an infalling observer one sees a normal black hole.

More specifically, probes with low energy

$$E \sim T_H$$

see the fuzzball and its distinct states.

For probes with $E \gg T_H$, the response of the fuzzball is different.

Roughly the idea is that, at the bottom of the fuzzball, we have many degrees of freedom. An infalling observer with energy $E \gg T_H$ excites these states in just the right way to produce an "effective interior".

[See papers on fuzzball complementarity for details.]

However one should be clear about the logic

The solutions are interesting but they **cannot represent typical states**

This is because of our previous stat-mech results.

Let $g_{\mu\nu}(x)$ be the metric operator at some point. Then we do expect that for typical states

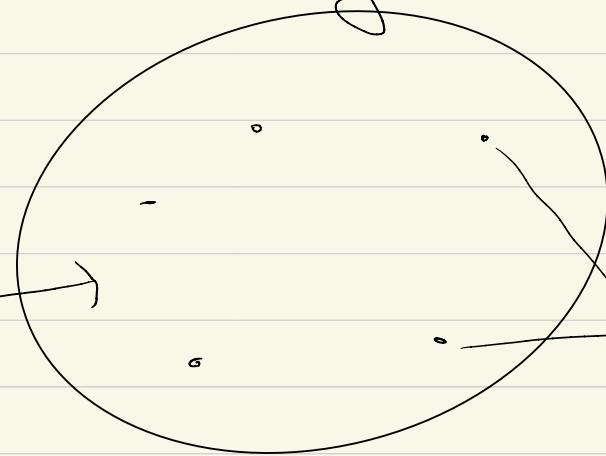
$$\langle \psi | g_{\mu\nu}(x) | \psi \rangle = \overset{\text{universal}}{g_{\mu\nu}(x)} + O(e^{-S/2})$$

But all the solutions vary at $O(1)$ from each other.

so the existence of solutions is interesting.
But they cannot be used to suggest
that **generic states** do not have a horizon.

The reason to invent the monogamy paradox
was to provide an **indirect argument**
and suggest that the interior does
not exist in generic states

Monogamy
paradox used
to argue that
generic states
are fuzzballs.



Cartoon of H-space

solutions are non-generic
states

Firewalls

There were several separate arguments given for firewalls.

Here we want to consider the AMPS argument.

This should be distinguished from the AMPSS arguments and the MP argument, which apply to large AdS black holes and not to evaporating bh.

↑
emphasize point in class

The First AMPS argument used precisely the monogamy paradox

is tantamount to cloning. For example [12], it violates strong subadditivity of the entropy,

$$S_{AB} + S_{BC} \geq S_B + S_{ABC} . \quad (2.3)$$

Let A be the early Hawking modes, B be our outgoing Hawking mode, and C be its interior partner mode. For an old black hole, the entropy is decreasing and so $S_{AB} < S_A$. The absence of infalling drama means that $S_{BC} = 0$ and so $S_{ABC} = S_A$. Subadditivity then gives $S_A \geq S_B + S_A$, which fails substantially since the density matrix for system B by itself is thermal. This argument is exactly as in Ref. [12], where the additional observations we are making are that i) the reasoning holds even under the postulates of black hole complementarity and ii) the modes b and c have high energy as seen by an infalling observer.

Actually, assuming the Page argument [18] the inequality is violated even more strongly:

Note that AMPS point out the argument is "exactly as in Ref. [12]"

But as pointed out here, Firewalls & fuzzballs are not quite the same as we will now see.

1) AMPS accept that the stat-mech arguments tell us that the metric **cannot** be modified significantly **outside** the horizon.

2) Also accept that **generic states must be featureless**

But now note that if the geometry outside is the same and there is no interior then we can compute

$$\langle T_{\mu\nu} \rangle.$$

near the horizon in such a state.

To compute this we usually need to compute

$$\lim_{x_1 \rightarrow x_2} \langle \partial_\mu \phi(x_1) \partial_\nu \phi(x_2) \rangle - \text{counterterms}$$

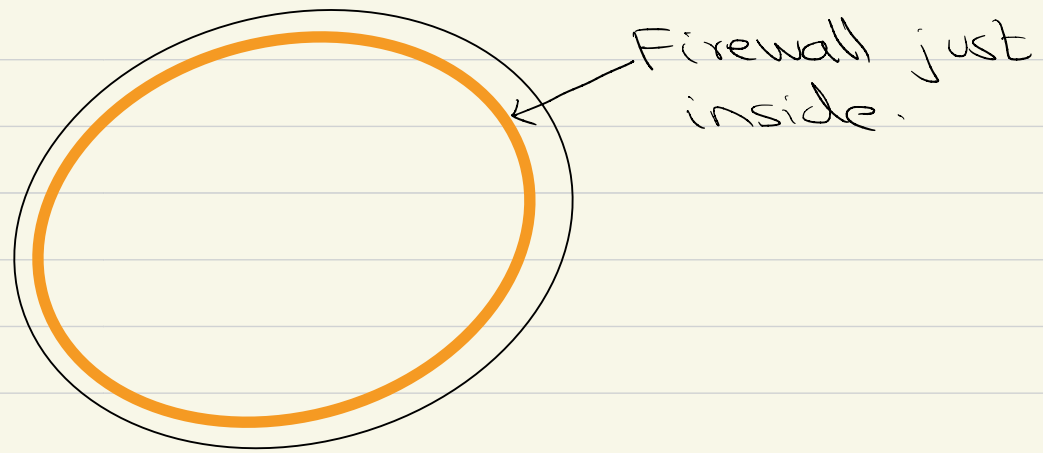
where the counterterms subtract the leading divergence obtained from

$$\lim_{x_1 \rightarrow x_2} \langle \phi(x_1) \phi(x_2) \rangle = \frac{N}{|x_1 - x_2|^{(d-1)/2}}$$

But the 2-pt function does not have the right form across the horizon in this proposal.

So $\langle T_{\mu\nu} \rangle$ diverges! Firewall!

So the picture is as follows



[Metric outside
exactly unchanged]

Questions:

a) Why is there such a large violation of EFFT ?

v) How is the Firewall formed and sustained?

Arguments for Firewalls are mostly indirect, and there is no direct mechanism for producing them.

" $A = R_1$ ", "Complementarity", " $ER = EPR$ "

The final set of proposals focus on a loophole in the monogamy paradox

Recall that the inequality

$$\langle C_{AB} \rangle^2 + \langle C_{AC} \rangle^2 \leq 8$$

only makes sense if A, B, C belong to different subsystems.

For instance if we set $B_1 = C_1$; $B_2 = C_2$ clearly we can get

$$\langle C_{AB} \rangle^2 + \langle C_{AC} \rangle^2 = 16$$

[Since $C_{AB} = C_{AC}$ in this case.]

But recall that to get

$$\langle C_{AB} \rangle^2 > 4$$

we needed $[B_1, B_2] \neq 0$

So now we will have $[B_1, C_2] \neq 0$

So a precise way to check whether we can apply the monogamy inequality is to see if the cross-commutators vanish.

In this case, the operators A, B were constructed in EFT.

We expect that

$$[A, B] \sim (G_N E)^{d-1} \sim \frac{1}{S}.$$

But we **never explicitly wrote down** C_1 and C_2

We simply argued that these operators should exist based on indirect arguments.

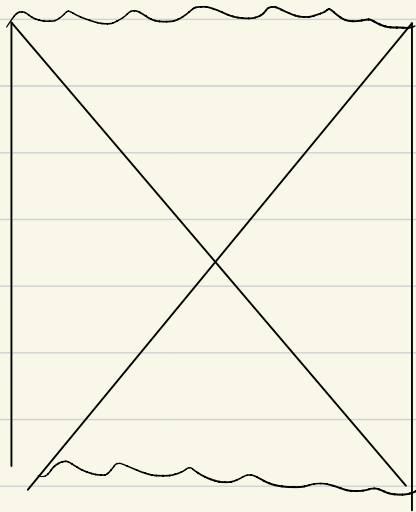
In bit-models, one can check that we expect C_1, C_2 to be extremely complicated.

For such operators one has to be careful about estimating $[B_i, C_j]$

The $A=R_v$ proposals all suggest that this commutator is large!

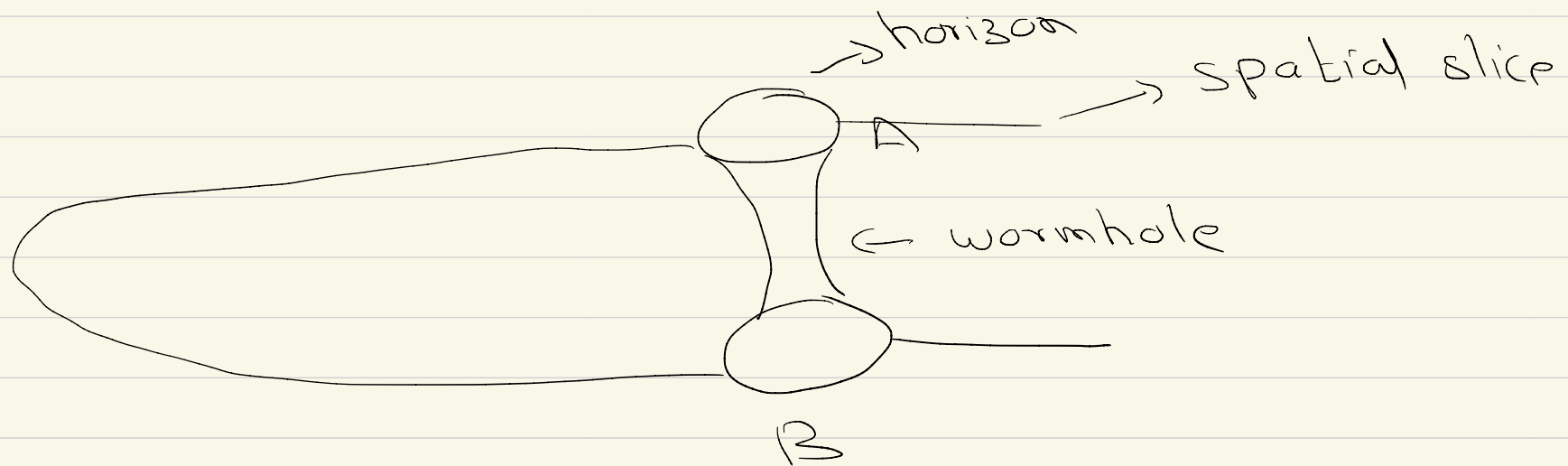
Therefore "the degrees of freedom in the interior are scrambled versions of dof far away."

The ER = EPR proposal is based on the same idea. In AdS/CFT we have the eternal black hole / thermofield duality



entangled CFTs dual to eternal l.h.

At eternal l.h. has two horizons
but the same interior. One way
to think of this is that we have
two very distant black holes,
connected by a wormhole.



The idea is that once the v.h. is maximally entangled with its radiation, one should think of the interior as being described by the radiation.

Note that $ER = EPR$ shouldn't be taken literally! Not all entangled qubits come with wormholes!

A better perspective might be that it just notes that thermofield eternal v.h. duality lends support to

$$A = R_v.$$

Criticisms

a) Are we violating "solar system physics"?

For a large l.h. the early radiation is **very far away**.

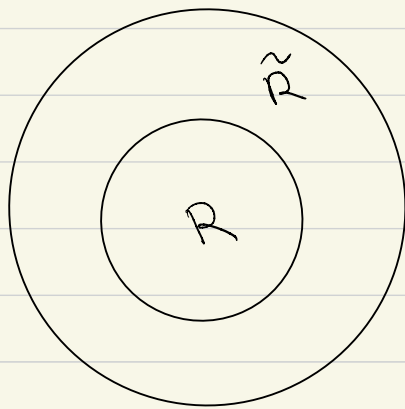
How can it affect the interior?

b) What is the origin of such effects? Is this a postulate of new physics?

In the next few lectures, we will formulate the

"principle of holography of information"

This states that in any theory of quantum gravity, a copy of the information available on a Cauchy slice is also available near the boundary.



← In such a configuration where $\tilde{R}$ surrounds R an observer in $\tilde{R}$ can use gravitational effects to extract info from R .

This is

- a) consistent with solar system physics!
- b) does not require new physics.

Arises from a careful consideration of the features of quantum mechanics and gravity.