

25-2-2020

Lecture 14: Holography of Information in Flat Space.

Let us now turn to the quantum theory in flat space

There is good news and bad news!

Good news

In one way, the operator algebra at $\mathcal{I}^+$ (or $\mathcal{I}^-$) is much simpler than at the boundary of AdS.

Due to the infinite volume of flat space, we can determine some commutation relations and constraints exactly, even in the full nonlinear theory.

For the news operators, we find the commutators

$$\begin{aligned} [N_{AB}(u, \Omega), N_{CD}(u', \Omega')] &= i 16\pi G \frac{1}{\sqrt{\gamma}} \delta^2(\Omega - \Omega') \delta(u - u') \\ &\quad \left[\gamma_{AC} \gamma_{DB} - \frac{1}{2} \gamma_{AB} \gamma_{CD} \right] \end{aligned}$$

The reason for this is that at large- r the nonlinear terms in the commutation relations die off!

Intuitively these commutators can be understood as follows.

The two independent components of C_{AB} behave like free-fields at every value of Ω .

So operators at different Ω commute.

We find that the Bondi mass obeys the constraint

$$\partial_u M(u) = - \int \sqrt{\gamma} d^2x \left[\frac{1}{8} N_{AB} N^{AB} + 4\pi G T_{uu}^{m,0} \right]$$

where

$$T_{uu}^{m,0} = \lim_{r \rightarrow \infty} r^2 T^{\text{matter}}$$

For the scalar field we have

$$T_{uu}^{m,0} = \frac{1}{2} \partial_u \phi \partial_u \phi$$

These commutation relations immediately give rise to a **Fock space** structure. To see this, just define

$$N_{AB}^{\pm}(\omega, \Omega) = \int e^{\pm i\omega u} N_{AB}(u, \Omega) du$$

Then we find that

$$\begin{aligned} & [N_{AB}^+(\omega, \Omega), N_{CD}^-(\omega', \Omega')] \\ &= i \frac{16\pi G}{\sqrt{\chi}} \delta(\Omega - \Omega') \left[\chi_{AC} \chi_{DB} - \frac{1}{2} \chi_{AB} \chi_{CD} \right] \\ & \times \int e^{i\omega u} e^{-i\omega' u'} \partial_u \delta(u - u') du du' \\ & \quad \Downarrow \\ & -i\omega \times 2\pi \delta(\omega - \omega'). \end{aligned}$$

So we see the commutation relations
of a free-field theory!

If we had a vacuum with the property
that

$$N_{AB}^+(\omega, \mathcal{R}) |0\rangle = 0 \quad \forall \omega \neq 0$$

← careful about this!

we could just generate excited states
by acting with

$$N_{AB}^-(\omega, \mathcal{R}) |0\rangle$$

To generate normalizable states

$$N(F) = \int \sqrt{\gamma} N_{AB}(\omega, \mathcal{R}) F^{AB}(\omega, \mathcal{R}) d\omega d^2\mathcal{R}$$

and consider

$$N(F_1) \dots N(F_n) |0\rangle$$

If there are other fields, say a scalar
with boundary value

we get a Fock space as

$$\text{span } \{ O(h_1) \dots O(h_n) N(F_1) \dots N(F_n) \} |0\rangle$$

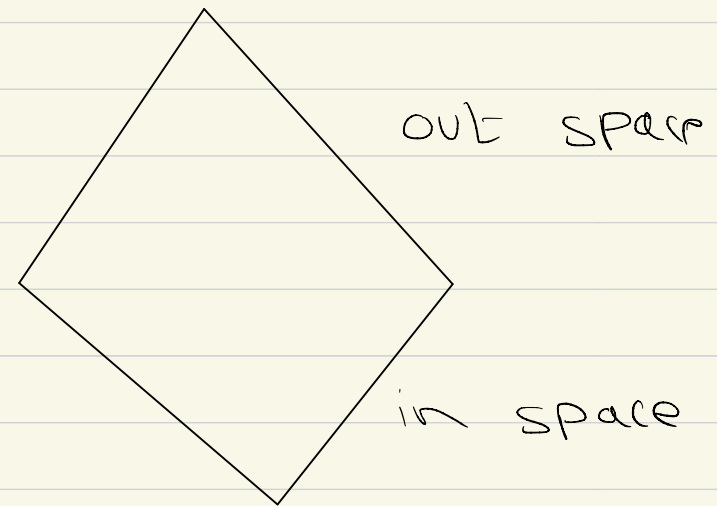
How did we get a Fock space?

1) You might be surprised that we found a Fock space description for the Full nonlinear theory. where is the dynamics?

2) But this is the "out" space

3) There is another complete description at g^-
→ the "in space"

4) The dynamics is all in the mapping of the in to out states, but either description **completely** specifies the state.



The Bad news

The bad news is that the vacuum is not unique but infinitely degenerate.

The vacua are labeled by their supertranslation charges.

$$Q_{\ell, m} = \frac{1}{4\pi G} \int \sqrt{\gamma} d^2\Omega \, m(u \rightarrow -\infty, \Omega) Y_{\ell, m}(\Omega)$$

Recall that in this notation, the Hamiltonian itself is $Q_{0,0}$

The Hamiltonian generates translations in u that are uniform on the celestial sphere.

Supertranslations are "angle-dependent" translations of $\mathcal{H}$.

Using the formalism of asymptotic quantization one can derive

$$[Q_{\ell,m}, C_{AB}(u, \mathcal{Z})] = Y_{\ell,m}(\mathcal{Z}) \partial_u C_{AB}(u, \mathcal{Z}) - 2 \left(D_A D_B Y_{\ell,m}(\mathcal{Z}) \right)^{TF}$$

[The inhomogeneous term involves the 0-mode of C and see 1401.7026 for a discussion on the factor of $2!$]

This is the action of the BMS supertranslation symmetry on g^+ .

So the distinct vacua are specified by giving a countably infinite sequence of numbers

← Notation for vacua

$$Q_{l,m} |\{s\}\rangle = S_{l,m} |\{s\}\rangle, [l \neq 0]$$

Supertranslation charges are often divided into a "hard" part and a "soft" part

$$Q_{l,m} = Q_{l,m}^{\text{hard}} + Q_{l,m}^{\text{soft}}$$

where the hard part is carried by excitations.

But the vacuum has no excitations! So

$$Q_{l,m} |\{s\}\rangle = Q_{l,m}^{\text{soft}} |\{s\}\rangle$$

These states are orthogonal

$$\langle \{s\} | \{s'\} \rangle = \delta(s-s')$$

and the action of hard operators does not change the soft charge.

So the full Hilbert space is a direct sum of Hilbert spaces built on top of these distinct vacua.

Define

$$H_s = \text{span} \{ O(h_1) \dots O(h_n) N(f_1) \dots N(f_n) \} | \{s\} \rangle$$

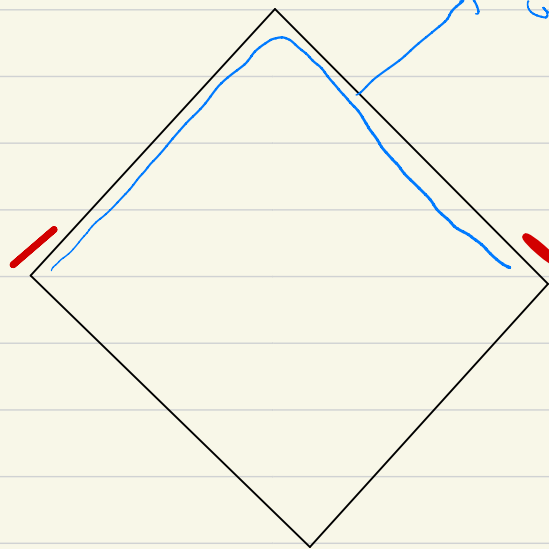
Then the full Hilbert space of massless particles is

$$H = \bigoplus_{\{s\}} H_s$$

Main result [Explain]

Any two distinct states in $\mathcal{H}$ can be distinguished by operators between $u = -\infty$ and $u = -\frac{1}{\epsilon}$ [in the vicinity of $\mathcal{I}_-^+$]

in QFT to identify states we need observations on all of $\mathcal{I}_-^+$



Define operator algebra $\mathcal{A}_{-\infty}$ as arbitrary polynomials of m, ϕ, ψ with $u \in [-\infty, -\frac{1}{\epsilon}]$

In gravity observations from $\mathcal{A}_{-\infty}$ are sufficient to identify states.

Proof follows 2 steps

1) Vacuum operators are in $A_{-\infty}$

We first want to show that

$$T_{S,S'} = | \{S\} \rangle \langle \{S'\} |$$

are in the algebra $A_{-\infty}$

[This is the part unique to gravity]

First recall that all the supertranslation charges and the Hamiltonian are elements of $A_{-\infty}$

$$H \in A_{-\infty}; \quad Q_{l,m} \in A_{-\infty}$$

[These operators are defined by integrals of $m(u, \Omega)$ in the limit $u \rightarrow -\infty$]

Recall if an operator is an element of an algebra, its spectral projectors are also in the algebra.

1) So the projector onto the manifold of vacua is an element of A_{-2}

$$P_0 = \int | \{ s \} \rangle \langle \{ s \} | \prod_{\ell, m} ds_{\ell, m}$$

2) Next we can perform a spectral decomposition of each supertranslation charge.

$$Q_{\ell, m} = \int ds \ s \ P_{\ell, m}[s]$$

So

$$P_0 \prod_{l,m} P_{l,m} [S_{l,m}] = |\{s\}\rangle \langle \{s\}| \in A_\infty$$

- 1) i.e. by projecting onto states of a supertranslation charge and then onto 0-energy, we project onto a given soft vacuum.
- 2) Both energy and supertranslation projections are required. [Explain]
- 3) We are using "projection" very sloppily since the vacua are δ -fn normalized. We need to smear a little to get true projectors.

Not only can we use operators in $A-\omega$ to select a given soft vacuum, we can also transition between different vacua.

This follows from:

$$P_0 e^{-\frac{i}{2} \int_{-\infty}^{-1/\epsilon} \sqrt{\gamma} C_{AB}(u, \Omega) G^{AB}(u, \Omega) d^2 \Omega} |\{s\}\rangle = |\{s'\}\rangle$$

where

$$S'_{lm} = S_{lm} + \int_{-\infty}^{-1/\epsilon} du d^2 \Omega \sqrt{\gamma} G^{AB}(u, \Omega) \left[D_A D_B - \frac{1}{2} \epsilon_{AB} \right] \gamma_{lm}$$

[Explain origin of formula. Comes from 0-mode commutator written previously]

So we have found that

$$T_{S,S'} = | \{S\} \rangle \langle \{S'\} | \in A_{-\infty}$$

All operators that act in the space of vacua can be found in $A_{-\infty}$.

[This is the physics that is unique to gravity]

Step 2: All operators $H \rightarrow \mathcal{H}$ are in $A-\infty$

Next we need to repeat the Reeh-Schlieder argument we made in AdS.

Recall that about any vacuum, we construct a Fock space as follows

$$\int_{-\infty}^{\infty} N_{AB}(u) F^{AB}(u) du \int_{-\infty}^{\infty} N_{A'B'}(u') F^{A'B'}(u') du \dots | \{S\} \rangle$$

↑
We have suppressed sphere directions.

But by the arguments we discussed previously

$$\text{span} [N(f_1) \dots N(f_n) \mid \{s\}]$$

$$= \text{span} [N(\tilde{f}_1) \dots N(\tilde{f}_n) \mid \{s\}].$$

where

f_i have support in $u \in (-\infty, \infty)$

$\tilde{f}_i$ have support in $u \in (-\infty, -\frac{1}{\varepsilon})$

Reminder of 1-particle proof.

$$\text{Let } |\psi\rangle = N(F) |\{s\}\rangle$$

$$\text{Assume } \langle \psi | N(\tilde{F}) |\{s\}\rangle = 0 \quad \text{for all } \tilde{F} \text{ with support in } u \in (-\infty, -\frac{1}{\varepsilon})$$

$$\Rightarrow \langle \psi | N_{AB}(u) |\{s\}\rangle = 0 \quad \forall u \in (-\infty, -\frac{1}{\varepsilon})$$

$$= \sum_E \langle \psi | E \rangle \langle E | N_{AB}(u) |\{s\}\rangle$$

$$= \sum_E \langle \psi | E \rangle \langle E | N_{AB}(0) |\{s\}\rangle e^{iEu}$$

This is analytic in the upper half u -plane

But then if it vanishes in

$$\langle \psi | N_{AB}(u) | f_s \rangle = 0 \quad \text{for } u \text{ in } (-\infty, -\frac{1}{\epsilon})$$

$$\Rightarrow \langle \psi | N_{AB}(u) | f_s \rangle = 0 \quad \forall u$$

But this is impossible since $|\psi\rangle = N(F) | f_s \rangle$

So $|\psi\rangle$ cannot exist.

Similar argument works for states formed by acting with higher polynomials in N

Putting everything together

Any operator $H \rightarrow H$ can be written as

$$A = \sum_{S, S'} c(n, m, S, S') |n_{\{S\}}\rangle \langle m_{\{S'\}}|$$

But

$$|n_{\{S\}}\rangle = X_n |\{S\}\rangle$$

$$|m_{\{S'\}}\rangle = X_m |\{S'\}\rangle$$

$$A = \sum_{\substack{n, m \\ s, s'}} X_n | \{s\} \rangle \langle \{s'\} | X_m^+$$

where $X_n, X_m \in A_{-\infty}$

So far this is true even in a LQFT

The operator in the middle is

$$T_{s, s'} = | \{s\} \rangle \langle \{s'\} |$$

$T_{s, s'} \in A_{-\infty}$ in gravity

So $A \in A_{-\infty}$ in gravity!
 [Algebra is closed under products and linear combinations.]

Discussion

Does this result hold if we consider states with massless excitations in the full theory?

We only required that

a) vacuum operators are in $\mathcal{A}_\infty$

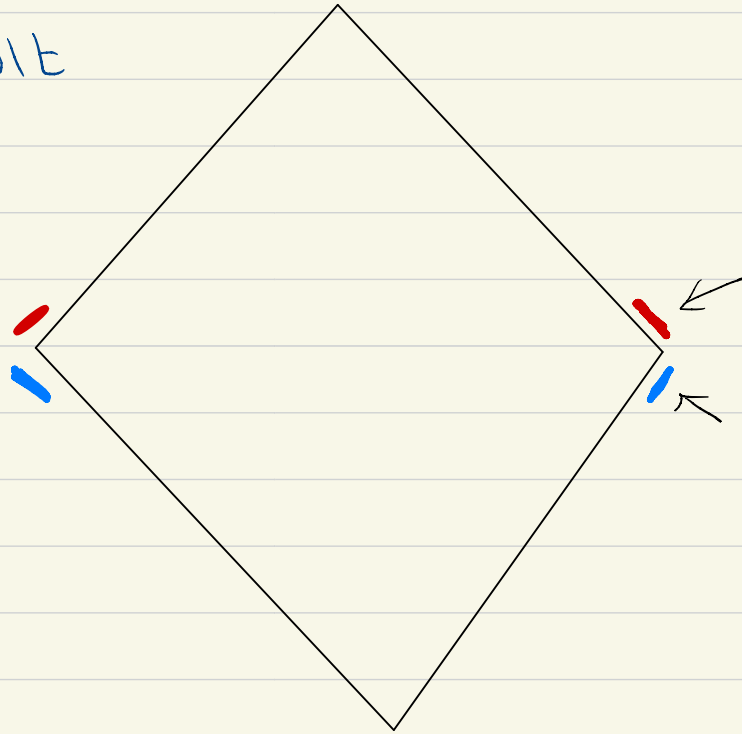
[Low-energy property so full theory should share this aspect of the low-energy theory.]

b) positivity of energy which allows a version of RS to hold.

Discussion

1) A similar result holds for

$g^- \rightarrow$ see Figure.



We showed info on g^+ is here

← Alternately info on g^- can be found here.

2) Comments on massive particles: trace out to obtain a mixed state on H

3) Comments on other dimensions.