

8 April 2021

Lecture 24: State Dependence

Yesterday we described how the mirror operators commute with ordinary operators inside simple correlators.

However, there is a subtlety.

Conserved charges

This is what we also argued that

$$[H, \tilde{a}_\omega] = \omega \tilde{a}_\omega$$

i.e. mirror operators do not commute with the Hamiltonian but carry the "wrong energy".

The idea is as follows.

Consider the states

$$|\psi\rangle, H|\psi\rangle, H^2|\psi\rangle, \dots, H^{D'}|\psi\rangle$$

where D' is a cutoff.

Each such state is an "equilibrium state" and we can define a little Hilbert space about such a state

$$H_{\psi} = \text{span} \{ A_{\text{sim}} H^n |\psi\rangle \}$$

These states are generically all linearly independent

Therefore we define on the direct sum of these spaces

$$\begin{aligned}\tilde{a}_\omega A_m H^n |\psi\rangle &= A_m e^{-\beta\omega/2} a_\omega^\dagger H^n |\psi\rangle \\ \tilde{a}_\omega^\dagger A_m H^n |\psi\rangle &= A_m e^{\beta\omega/2} a_\omega H^n |\psi\rangle\end{aligned}$$

In general

$$\tilde{A}_q A_m H^n |\psi\rangle = A_m e^{-\beta H/2} A_q^\dagger e^{\beta H/2} H^n |\psi\rangle$$

Let us check that this gives the right commutators

$$[\tilde{a}_\omega, H] A_n |\psi\rangle$$

$$= \tilde{a}_\omega H A_n |\psi\rangle - H \tilde{a}_\omega A_n |\psi\rangle$$

$$= \tilde{a}_\omega [H, A_n] |\psi\rangle + \tilde{a}_\omega A_n H |\psi\rangle - H A_n \tilde{a}_\omega |\psi\rangle$$

$$= -i \tilde{a}_\omega \dot{A}_n |\psi\rangle + A_n \tilde{a}_\omega H |\psi\rangle - A_n H \tilde{a}_\omega |\psi\rangle$$

Here $\dot{A}_n \equiv [H, A_n]$ is assumed to be part of the set of simple operators $+ [A_n, H] \tilde{a}_\omega |\psi\rangle$

[obvious if A_n is made up of simple polynomials of $a_\omega, a_\omega^\dagger$]

The last term also gives

$$[A_n, H] \tilde{a}_\omega |\psi\rangle = i \dot{A}_n \tilde{a}_\omega |\psi\rangle$$

and from the first term

$$-i \tilde{a}_\omega \dot{A}_n |\psi\rangle = -i \dot{A}_n \tilde{a}_\omega |\psi\rangle$$

So we are left with

$$A_n \tilde{a}_\omega H |\psi\rangle - A_n H \tilde{a}_\omega |\psi\rangle$$

$$= e^{-\beta\omega/2} [A_n a_\omega^+ H |\psi\rangle - A_n H a_\omega^+ |\psi\rangle]$$

$$= -\omega e^{-\beta\omega/2} A_n a_\omega^+ |\psi\rangle$$

$$= -\omega \tilde{a}_\omega A_n |\psi\rangle; \text{ Precisely as }$$

expected.

Near equilibrium states

The Frozen vacuum problem

So far we have defined the mirror operators on equilibrium states. But this definition cannot be the right one for all states

For instance, consider

$$|\psi_{ne}\rangle = U |\psi\rangle$$

where $|\psi\rangle$ is equilibrium and U is a simple unitary.

Then consider

$$\tilde{a}_\omega |\psi_{ne}\rangle$$

We know that

$$\tilde{a}_\omega U |\psi\rangle = e^{-\beta\omega/2} U a_\omega^\dagger |\psi\rangle$$

But if we used

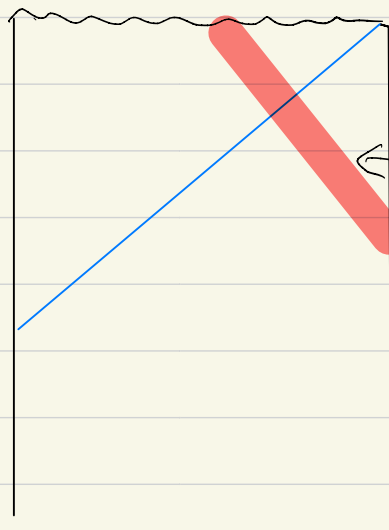
$$\tilde{a}_\omega |\psi_{ne}\rangle = e^{-\beta\omega/2} a_\omega^\dagger |\psi_{ne}\rangle$$

This would give the wrong answer

if $[a_\omega^\dagger, U] \neq 0$

This is sometimes called the "Frozen vacuum" problem.

The operator U is like an excitation made after the bh has settled down



additional excitation sent after the black hole forms that intersects the infalling observer.

$\mathcal{F} \tilde{a}_\omega$ "behave" the same way in $U|\psi\rangle$ as they do in $|\psi\rangle$, that

would not allow such processes and "freeze" the horizon in a vacuum-like state.

This is called the Frozen vacuum problem.

Therefore, if we are "presented" with the state $|\psi_{n.e}\rangle$, we need to be able to define mirror operators that have the correct behaviour.

The point is that for states of the form $|\psi_{ne}\rangle$, we can

back-calculate the base state and also the unitary excitation.

We look for V such that

$$\langle \psi_{ne} | V A_p(t) V^\dagger | \psi_{ne} \rangle$$

is time-independent. For all $A_p(t)$

It is clear that only $V = U$ works

This is because if we take $V = U W^\dagger$ then

$$V^\dagger |\psi_{ne}\rangle = W |\psi\rangle$$

and we can find some operator A_p so that

$$\langle \psi | w^\dagger A_p(E) w | \psi \rangle$$

is not time-independent.

For near-equilibrium states, we define.

$$\tilde{a}_w A_m |\psi_{ne}\rangle = A_m e^{-\beta w/2} V a_w^\dagger V^\dagger |\psi_{ne}\rangle$$

$$\tilde{a}_w^\dagger A_m |\psi_{ne}\rangle = A_m e^{\beta w/2} V a_w V^\dagger |\psi_{ne}\rangle$$

The first factor of $V^\dagger$ "strips off" the unitary that creates the excitation, puts the $a_w^\dagger$ next to the equilibrium state.

The second factor reinserts the unitary.

So far we have described the mirror operator construction in specific equilibrium and near-equilibrium states.

Say we have one such state and we construct mirror operators using the procedure above.

Recall that with $|V_m\rangle = A_m |\psi\rangle$, $|U_m\rangle = A_m e^{-\beta\omega/2} a_\omega^\dagger |\psi\rangle$

We define

$$\tilde{a}_\omega^{\psi_1} = \sum_{mn} g^{mn} |U_m\rangle \langle V_n|$$

This is not a state-dependent operator.

As defined, if we act this operator on some different equilibrium state $|\psi_2\rangle$,

$$\tilde{Q}_w^{\psi_1} |\psi_2\rangle = O(\langle\psi_1|\psi_2\rangle) = O(e^{-S/2})$$

[Explain: even $|\psi_1\rangle\langle\psi_1|$ is a state-dependent operator.]

It is well defined on the whole $\mathcal{H}$ -space and it just happens to give 0 on states orthogonal to $|\psi_1\rangle$.

So we could try and improve our definition.

Perhaps we try

$$\tilde{a}_\omega^{\psi_1} + \tilde{a}_\omega^{\psi_2}$$

Now we have an operator that can describe the experience of an infalling observer in $|\psi_1\rangle$, $|\psi_2\rangle$ but also in

$$|\psi\rangle = a|\psi_1\rangle + b|\psi_2\rangle$$

This is because

$$\begin{aligned} & \langle \psi | \tilde{a}_\omega a_\omega | \psi \rangle \\ &= |a|^2 \langle \psi_1 | \tilde{a}_\omega a_\omega | \psi \rangle + |b|^2 \langle \psi_2 | \tilde{a}_\omega a_\omega | \psi \rangle \\ &+ a^* b \langle \psi_1 | \tilde{a}_\omega a_\omega | \psi_2 \rangle + b^* a \langle \psi_2 | \tilde{a}_\omega a_\omega | \psi_1 \rangle \end{aligned}$$

But in general, notice that cross-correlators of simple operators in two generic equilibrium states are highly suppressed

$$\langle \psi_1 | A_1 A_2 | \psi_2 \rangle = O(e^{-S/2})$$

So the superposition of two states behaves like a classical mixture

[Explain that $a^\dagger$ annihilates states outside H_{ψ_1}]

So now we have mirror operators that work in

$$H_{\psi_1} \oplus H_{\psi_2}.$$

Similarly if $|\psi_1\rangle \dots |\psi_n\rangle$ are equilibrium microstates we could build up larger and larger sums

$$\tilde{a}^{\psi_1} + \tilde{a}^{\psi_2} + \dots + \tilde{a}^{\psi_n}$$

when n reaches
 $n \sim O(e^S)$

we might imagine that we have a mirror operator that works everywhere in the Hilbert space.

But now the cross terms become important!

Consider inserting the operator back in the state $|\psi_1\rangle$.

Then

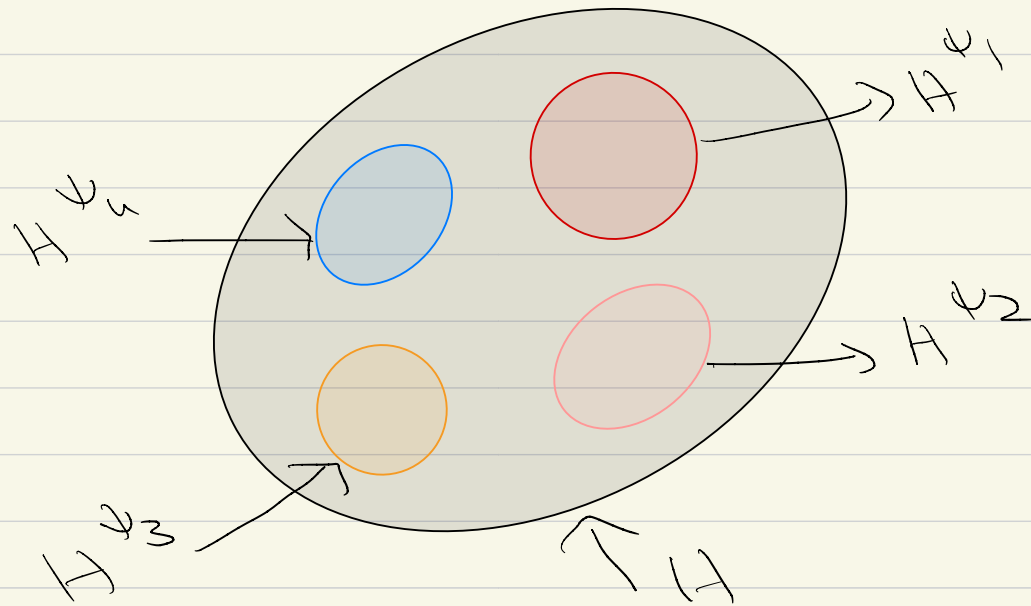
$$\langle \psi_1 | \tilde{a}_\omega a_\omega | \psi_1 \rangle = \langle \psi_1 | \tilde{a}_\omega^{\psi_1} a_\omega | \psi_1 \rangle + \sum_{i=2}^n \langle \psi_1 | \tilde{a}_\omega^{\psi_i} a_\omega | \psi_1 \rangle$$

When $n = O(e^S)$, the second term can start competing with the first term.

But the first term already gives the right expected answer for the correlator!

So instead of getting an operator that works in all states, we might get an operator that works in no state!

Pictorially one can imagine the following



Each of these operators is defined on a little Hilbert space.

If we start adding too many of them, we get conflicts where the spaces overlap.

a) It might happen that with the right definition, the operators mesh smoothly at the intersection of the spaces.

b) Or it might happen, that no operator can be found that works everywhere on the H space.

The paradoxes we found for large black holes tell us that we cannot find an operator $\hat{a}_w$ that works globally.

At most we can find an operator that works in an exponentially small fraction of states.

[Aside: as emphasized, it is not

clear if any such problem occurs for black holes in flat space because such black holes are exponentially unlikely states

$$\left. \begin{aligned} S_{\text{Hawking-rad}} &> S_{\text{b.h.}} \\ \Rightarrow e^{S_{\text{Hawking-rad}}} &\gg e^{S_{\text{b.h.}}} \end{aligned} \right\}$$

The idea of state-dependence is as follows

We abandon the idea of trying to find a global mirror operator.

Instead, we declare:

"The infalling observer in the state $|\psi\rangle$, automatically uses the operators that act correctly within $H|\psi\rangle$ "

"the physical operators used depend on the state."

Why is this unusual?

Usually in quantum mechanics, we fix a basis of observables ahead of time



The observer can measure O in different states

If we ask: what is the probability a state has some feature (eg. firewall / smooth horizon) this is given by expectation value of a projector $\langle \psi | P | \psi \rangle$.

Why is state dependence conceivable

For the infalling observer, observer is part of the system.

So perhaps the basis of observables is not set a priori

The observer uses the "natural local observables" available at the horizon.

This may depend on the microstate.

Also note that this state-dependence is not visible directly to the observer.

If the observer measures

$$\langle \psi | A_n \tilde{a}_w A_m | \psi \rangle$$

[correlator with H^ψ]

Then $\tilde{a}_w$ is fixed. The state dependence only arises when one makes a significant change and changes the microstate to

$$|\psi'\rangle \notin H^\psi.$$

Infalling observer sees ordinary Q.M.