21 Jan 2021 Lecture 4

Relationship of global modes to near-horizon modes.

We already mentioned that, in Kruskal coordinates, the metric becomes

$$ds^2 \longrightarrow -dU\,dV + r^2 d\Omega_{d-1}^2 + \ldots$$

near the horizon

So we can define "near-horizon" modes as in lectures 1 and 2.

$$a_{nh} = \frac{r_0^{d-1}}{\sqrt{\pi \omega_0}} \int \partial_U \phi(U, V=0, \Omega)(-U)^{-i\omega_0} T(-U)\, dU\, Y_l^*(\Omega)\, d\Omega$$

note opposite convention

$$\tilde{a}_{nh} = \frac{r_0^{d-1}}{\sqrt{\pi \omega_0}} \int \partial_U \phi(U, V=-\epsilon, \Omega)\, U^{i\omega_0} T(U)\, Y_l(\Omega)\, d\Omega$$

We can do the integral carefully.

But let us guess the answer for $a_{nh}$ in terms of $A_{\omega,l}$ and $B_{\omega,l}$

We already argued that near the horizon, the modes $A_{\omega,l}$ are multiplying

$$A_{\omega,l}\, e^{-i\omega t}\left(e^{i\omega r_*} + g_{\omega,l}\, e^{-i\omega r_*}\right).$$

$$\sim A_{\omega,l}\left(U^{i\omega/\kappa} + g_{\omega,l}\, V^{-i\omega/\kappa}\right)$$ [up to constants]

$B_{\omega,l}$ multiply

$$B_{\omega,l}\, e^{-i\omega t}\, e^{-i\omega r_*}$$

$$\sim B_{\omega,l}\, V^{-i\omega/\kappa}$$

[up to constants]

So the integral for $a_{nh}$ only picks up the $A_{\omega,l}$ modes. "$l$" is fixed by angular integral

But moreover since the integral is over a large range of $\log u$ the contribution is sharply peaked from

$$\omega = k\omega_0$$

So we can write

$$a_{nh} = a_{\omega_0 k, l}$$

where

$$a_{\omega,l} = \int A_{\omega',l}\, q(\omega',\omega)\, d\omega'$$

In principle we can work out "$q$" carefully.
But we don't need to!

We only need

1) $q$ is sharply peaked about $\omega' = \omega$

2) After smearing, the $a_{\omega,l}$ modes are correctly normalized

$$[a_{\omega,l}, a^{\dagger}_{\omega,l}] = 1$$

We know this even without working out $q$ because the near horizon modes always obey these relations which come from the canonical commutators of the field.

Now we immediately see that

$$\langle a_{\omega,l}\, a^{\dagger}_{\omega,l} \rangle = \frac{1}{1 - e^{-2\pi \omega_0}}$$

But $\omega_0 = \omega / \kappa$

with $\beta = \dfrac{2\pi}{\kappa}$ we have

$$\boxed{\langle a_{\omega,l}\, a^{\dagger}_{\omega,l} \rangle = \frac{1}{1 - e^{-\beta\omega}}}$$

Hawking radiation!

So

1) The existence of a smooth horizon

2) The late-time emergent $t \to t + \delta t$ isometry

force the $a_{\omega,l}$ modes to be thermally occupied

Since $a_{\omega,l}$ is prop to $A_{\omega,l}$ and since $A_{\omega,l}$ is outgoing at $r_* \to \infty$ we see this leads to an outgoing flux

The precise stress-tensor far away depends on the greybody factors and precise factors between $a_{\omega,l}$ and $A_{\omega,l}$.

But we will not need these in detail.

Note the derivation does not constrain the $B_{\omega,l}$ modes.

a) If we put $B_{\omega,l}$ in vacuum,

called "Unruh state"

b) If we populate $B_{\omega,l}$ thermally

called "Kruskal state"

Other choices are possible for $B_{\omega,l}$.

# Hawking radiation in Ads.

We now repeat the analysis in Ads, which is very similar. We will just point out some differences.

We will consistently work in spacetimes that are asymptotically global Ads

$$ds^2 \xrightarrow[r \to \infty]{} -(1+r^2)\,dt^2 + \frac{dr^2}{1+r^2} + r^2\,d\Omega^2_{d-1}$$

We have set the AdS radius to 1.

We want to consider black holes formed from collapse that, at late times, tend to AdS - Schwarzschild

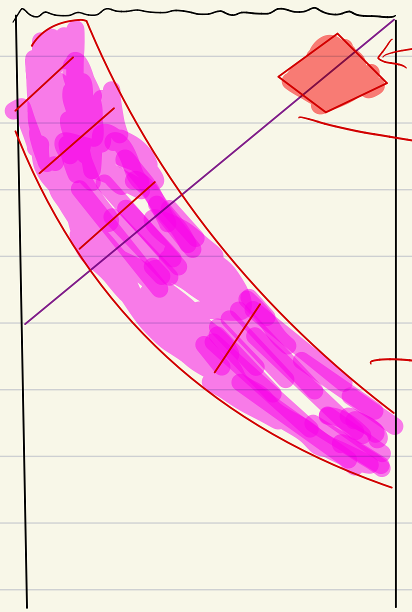
$$ds^2 \xrightarrow[t \gg 1]{} -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\, d\Omega_{d-1}^2$$

Here

$$f(r) = 1 + r^2 - \frac{\mu}{r^{d-2}}$$

The horizon is at $r = r_h$ where

$$f(r_h) = 0 .$$




The Penrose diagram of the spacetime looks like the figure above.

We are interested in times where perturbations due to infalling matter have decayed but not times that scale (in AdS units) like the B.H. entropy.

We are always interested in fields with normalizable boundary conditions.

$$\phi \to 0$$

as $r \to \infty$

This corresponds to a system that is not disturbed by external sources and evolves autonomously.

Emphasize point: sources are not required to extract any bulk physics.

For fields of mass $m$, it turns out that

$$\phi \to \frac{1}{r^{\Delta}} \quad \text{as } r \to \infty$$

where $$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2}$$

In AdS/CFT the field is dual to an operator of dimension $\Delta$ in the CFT.

As usual we introduce tortoise coordinates

$$dr_* = \frac{dr}{f(r)}$$

Near the horizon $r_* \to -\infty$

Near the boundary, $r_* \to$ constant

Lets consider the expansion of the field

Again, consider a field mass $m$.

Lets expand the field as

$$\phi = \int d\omega \, A_{\omega,\ell} \, f^{st}(\omega,\ell,r_*) \, e^{-i\omega t} \, Y_\ell(\Omega) + h.c.$$

But now only "standing wave" solutions are allowed.

difference from flat space

Near $r_* \to -\infty$

$$f_{st}(l, \omega, r_*) \to e^{i\omega r_*} + \underline{e^{-i\delta_{\omega,l}}}\, e^{-i\omega r_*}$$

$g_{\omega,l} \to$ relative coefficient has to be a phase.

The canonical commutators

$$r^{d-1}[\phi, \dot{\phi}] = i\delta(r_* - r_*')\,\hat{\delta}(\Omega, \Omega')$$

Consider the $\omega$-integral near $r = r_h$

We need

$$\int i\omega\, r^{d-1}\, [A_{\omega,\ell}, A^{\dagger}_{\omega',\ell}] \left( e^{i\omega r_* - \omega' r'_*} + g_{\omega,\ell}\, g^{*}_{\omega',\ell}\, e^{-i\omega r_* + \omega' r'_*} \right) d\omega$$

only +ve

$$\propto \delta(r_* - r'_*)$$

[We have not displayed terms linear in $g_{\omega,\ell}$ since they cannot contribute to $\delta(r_* - r'_*)$]

requires $|g_{\omega,\ell}|^2 = 1$, with

$$[A_{\omega,\ell}, A^{\dagger}_{\omega',\ell}] \propto \delta(\omega - \omega')$$

Near the boundary, $r \to \infty$

$$f_{st}(\omega, \ell, r_*) \longrightarrow \frac{\sqrt{G_{\omega,\ell}}}{r^{\Delta}}$$

dual operator

and $\quad O_{\omega,\ell} = \sqrt{G_{\omega,\ell}}\, A_{\omega,\ell}$.

We can also quantize the field behind the horizon

Behind the horizon we write

$$\phi = \int d\omega \left[ A_{\omega,l} e^{-i\delta_{\omega,l}} e^{-i\omega t} Y_l(\Omega) \tilde{f}_{st}(\omega, l, r_*) + \tilde{A}_{\omega,l} e^{i\omega t} Y_l^*(\Omega) \right]$$

where

$$\tilde{f}_{st}(\omega, l, r_*) \xrightarrow[r_* \to -\infty]{} e^{-i\omega r_*}$$

and $\tilde{A}_{\omega,l}$ are new modes not necessarily simply related to $O_{\omega,l}$.

We now introduce near-horizon modes as previously. We find

$$a_{nh} = a_{\omega_0 \kappa, l}$$

$$\tilde{a}_{nh} = \tilde{a}_{\omega_0 \kappa, l}$$

where $\kappa = f'(r_h)/2$

$$a_{\omega, l} = \int A_{\omega', l} \, q_l(\omega, \omega') \, d\omega'$$

$$\tilde{a}_{\omega, l} = \int \tilde{A}_{\omega', l} \, \tilde{q}_l(\omega, \omega') \, d\omega'$$

and $q, \tilde{q}$ are sharply peaked around $\omega = \omega'$

The normalization is again fixed by

$$[a_{\omega,l}, a^{\dagger}_{\omega,l}] = 1.$$

Note that although the $a_{\omega,l}$ are thermally occupied, there is no flux at $r \to \infty$ since the modes are standing waves.

So AdS bhs do not evaporate with these boundary condition. Instead we have a bh surrounded by a thermal atmosphere.