3 Feb 2021

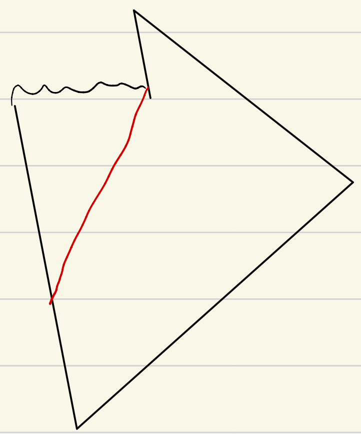
Lecture 7: Resolving Hawking's Paradox

Hawking's original argument for information loss was based on

a) a concrete computation: the out states are occupied as

$$\langle a_{\omega,\ell} \, a^{\dagger}_{\omega,\ell} \rangle = \frac{1}{1-e^{-\beta\omega}}$$

b) Intuition from the Penrose diagram



The region in the interior appears to be a "hidden surface" and Hawking suggested that observers at $\mathcal{I}^+$ should adopt a "principle of ignorance" about this surface.

Last time we emphasized that in a large enough Hilbert space, almost all states look thermal

$$\left| \langle\psi|A|\psi\rangle - \frac{1}{e^S}\operatorname{tr}(A) \right| \sim e^{-S/2}, \text{ for typical } |\psi\rangle.$$

(See previous lecture for defn of "typical" and a more precise result.)

So Hawking's calculation is far from being sufficient to lead to a paradox

The natural perturbative parameter is

$$\underbrace{G_N T^{d-1}}_{\text{DIM ANALYSIS}} \sim \frac{1}{S}$$

So to obtain a true paradox, one would have to repeat Hawking's calculation to all orders in perturbation theory and include nonperturbative effects.

Even the leading order computations have not been done

What about intuition from the 'principle of ignorance?'

First lets consider the no hair theorem.

This suggests that at late times, the black hole is characterized only by its mass and conserved charges.

Sometimes people use this as an argument for information loss.

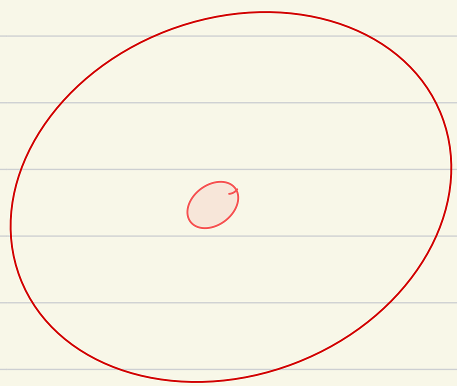
However, the no-hair theorem is only a **classical result**. It does not hold quantum mechanically!

This is **not a technical issue of lack of proof**

One should never expect a quantum no-hair theorem for the following reasons.

Recall that in a theory of gravity on can mease the energy far away by virtue of the Gauss law.

$$H = \frac{1}{16\pi}\int \left(\partial^i h_{ij} - \partial_j h_{ii}\right) \cdot n^j \, d^2S$$



This integral is defined at $r \to \infty$. So the energy of a state can be measured at $r \to \infty$.

Now say that we have a theory of quantum gravity in which a black hole state is

$$|\psi\rangle = \sum a_i \, |E_i\rangle$$

[Explain why spread in energy is necessary. → dynamics.]

for some coeffs a_i

The observation far away should give us "H".
But, in q.m., we can not only measure

$$\langle \psi | H | \psi \rangle = \sum |a_i|^2 E_i$$

We can also measure

$$\langle \psi | H^2 | \psi \rangle = \sum |a_i|^2 E_i^2$$

Note that

$$\langle \psi | H^2 | \psi \rangle \neq \langle \psi | H | \psi \rangle^2$$

so, already in quantum mechanics we have information about moments of the charges which we do not have classically.

In fact we can ask about the probability of getting an answer between E and $E+\Delta$ upon an energy measurement, which is

$$p(E, E+\Delta) = \sum_{E < E_i < E+\Delta} |a_i|^2$$

So we immediately see that the black hole state cannot be characterized just by its energy! We must **at least also have** moments of the energy

But, there is more. If O is some observable in the exterior, the quantum-mechanically, we also have

$$\langle H O \rangle \neq \langle H \rangle \langle O \rangle$$

So if someone wanted to prove an analogue of the no-hair theorem, they would have to prove

$$\langle H O \rangle \xrightarrow[\text{late times}]{} \text{approaches universal value.}$$

Moreover, they would have to show this to an accuracy better than $e^{-S/2}$ since otherwise these exp small tails would preserve information.

Nothing of this kind has even been attempted

There is a deeper reason to be skeptical of any such efforts to extend the no-hair theorem.

For any such theorem to hold we would

like to be able to change the state of the system in the interior (red part of Cauchy slice) **without modifying** the exterior [only if one can do this can the exterior observer not know about the interior.]

So to prove anything like a no-hair theorem we would like to find a unitary operator U that commutes with all operators on the green part of the slice but modifies the state of the red part

$$\langle \psi | U^{\dagger} O(\text{out}) U | \psi \rangle = O.$$

In particular since H is an operator outside, we must have

$$U^\dagger H U = H$$

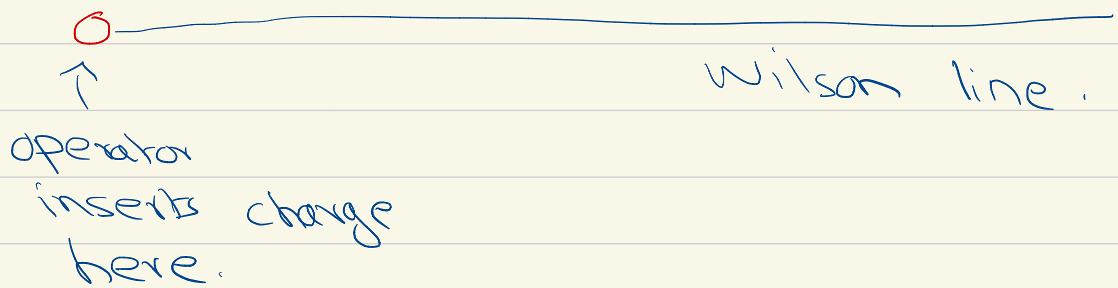
But an operator with 0-energy cannot be localized in a compact region by the uncertainty principle!

[If the operator creates an excitation with position-uncertainty Δx, its momentum uncertainty $\sim \frac{1}{\Delta x}$.]

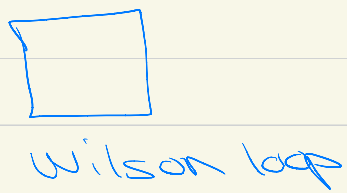
This <u>is not</u> a proof but already suggests we cannot find an operator that only changes the interior without changing the exterior.

There is another way to understand this issue with an analogy with gauge theory.

In gauge theory, an operator that inserts a charge at a location must be dressed using a Wilson line



In gauge theories we can construct gauge invariant operators that carry no charge and are localized



But in gravity, any excitation in the bh-interior must carry energy and so the operator that creates it must be dressed via a Wilson line that runs off to infinity. [There are no -ve charges in gravity]

This is sometimes stated as

" $\nexists$ no local gauge-invariant operators in gravity"

Possible confusion: isn't the Ricci scalar $R(x)$ a local gauge-invariant op?

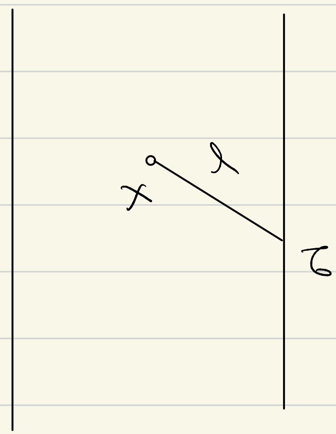
No, because under a diff

$$x^\mu \to x^\mu + \varepsilon^\mu$$

$$R(x) \to R(x) + \partial_\mu R(x)\,\varepsilon^\mu$$

To make the operator gauge-invariant, we need a way of specifing the position "x" in a way that doesn't change under diffs.

This can be done by specifying the position 'relationally', by shooting geodesics from infinity



Pos^n in AdS specified by shooting a geodesic from a bdry pt.

But any such procedure makes the operator "extend to infinity"

So once again we seem to find

"there is no way to change the state of the interior without changing exterior."

For these reasons, we will discuss later in the course that

Hawking's principle of ignorance

gets replaced by its converse

principle of holography of information

But, for now, we only need to note that

the principle of ignorance cannot be used to frame any precise paradox.

"What is the mistake in Hawking's argument?"

a) It is incorrect to argue, based on low-pt correlators of outgoing fields, that the final state is thermal.

b) It is incorrect to use the no hair theorem or classical intuition to argue that the observer outside has no information about the interior.