Quantum Aspects of Black Holes - Lec 20 - 8 Nov 2016

Last time we discussed the Page curve.

[review briefly]

This curve is derived by considering a nm dimensional Hilbert space with $m < n$ and considering its division into a n-dim H-space and a m-dim H-space.

In a generic state, we showed

$$S_m = -\operatorname{tr}(\rho_m \ln \rho_m) = \ln m - O\left(\frac{m}{n}\right)$$

One way to think of the decreasing S_m in the latter part of the curve is in terms of "information coming out" of the B.H.

Specifically, imagine that we have no limits on our observational ability [we will revisit this issue later]

Then while we have access to less than $e^{S/2}$ degrees of freedom, the density matrix is very close to the identity matrix and we can say little about the state of the full system.

But beyond this point, we do get information

If the Hawking radiation encompasses

$e^{S/2 + T}$

degrees of freedom, our previous analysis shows the density matrix is $1 \cdot e^{-S/2+T}$ on a

$e^{S/2-T}$ dim space.

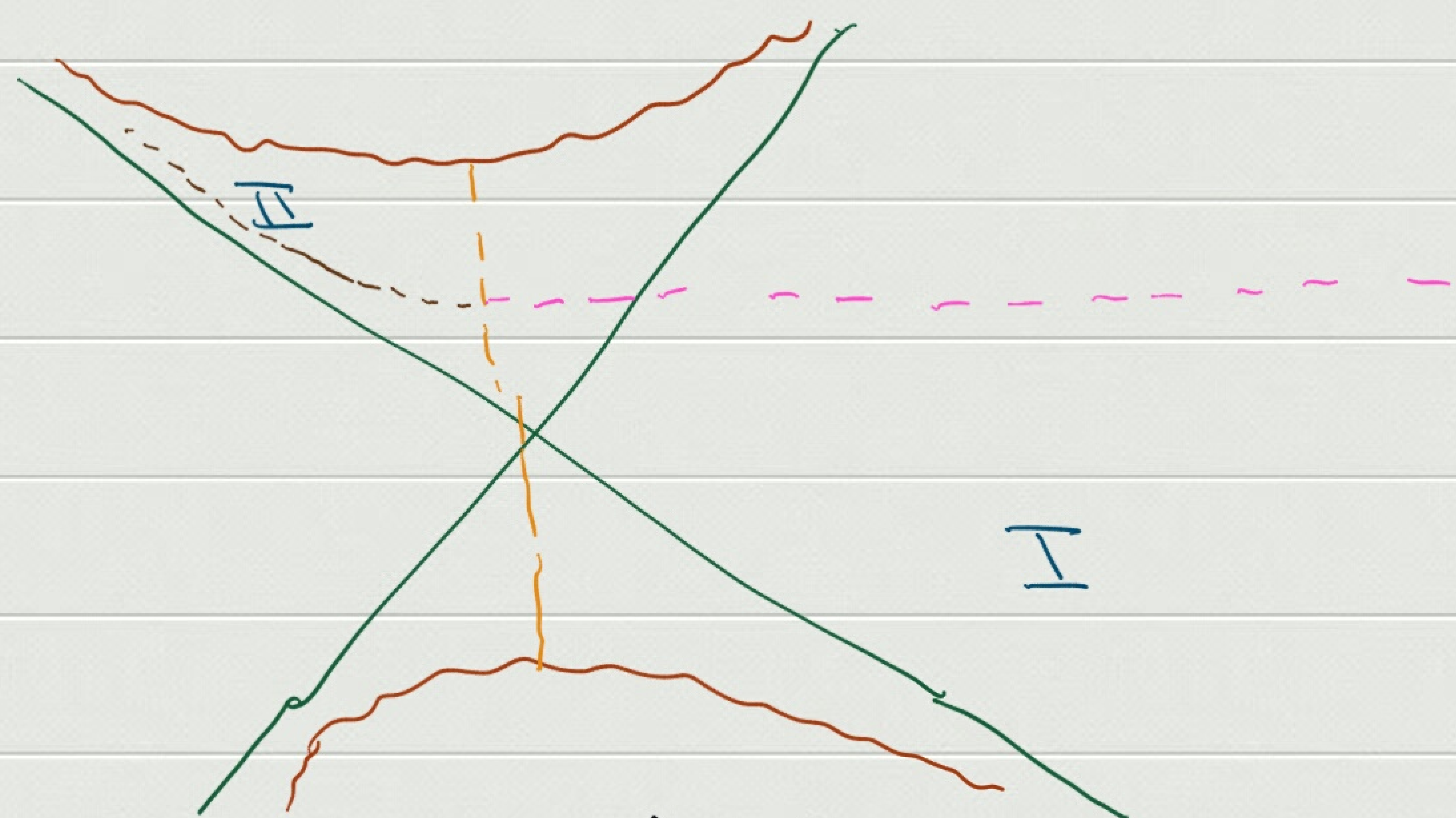
Which $e^{S/2-T}$ dim space is entangled with the b.h. gives us information about the initial state.

To refine the information paradox, we need one additional notion → the notion of nice slices.

Nice slices are Cauchy slices with small curvature everywhere, and which stay away from the singularity at all points.

It is quite easy to construct such slices. These slices allow us to argue that effective field theory should be valid everywhere on them with no large corrections due to quantum gravity.

For example in the eternal black hole geometry we can construct slices that foliate regions I & II as follows



Recall the U, V coordinates.

In region II, we consider the slice $UV = R^2$

Recall that in region II, $U = e^{(r_* - t)/4m}$ $V = e^{(r_* + t)/4m}$
and $r_* = r + 2m \ln \left| \frac{r-2m}{2m} \right|$

so $UV = R^2 \Rightarrow e^{r_*/2m} = R^2$
or
$$r_* = 2m \ln R^2$$

So by choosing $R^2 = O(1)$ we can keep r_* well away from 0 — where the singularity is.
This is the brown hyperbola in the figure

The vertical orange line in the middle is the line where $V = 0$ $(t=0)$. Beyond this line (i.e. for $V > U$)

Beyond this line we consider

$$V + U = 2R.$$

Recall that outside the horizon we have

$$U = -e^{(r_* - t)/4m}$$

so the relation above becomes

$$e^{r_*}\left(e^{t/4m} - e^{-t/4m}\right) = 2R$$

As r_* becomes large this forces us to $t=0$.
So this slice asymptotes to $t=0$

We can now construct a family of nice slices that asymptote to $t=\tau$ by simply substituting $U \to U e^{\tau/4m}$; $V \to V e^{-\tau/4m}$ above.

i.e. we have

$$UV = R^2 \, , \text{ for } V e^{-\tau/4m} > V e^{+\tau/4m}$$

$$U e^{\tau/4m} + V e^{-\tau/4m} = 2R$$

These slices are shown below

To be very strict we can also smoothen out the "junction" between the two patches of the slice.

We can draw these nice slices for black-holes formed from collapse as well

Let us note two funny aspects about the nice slices

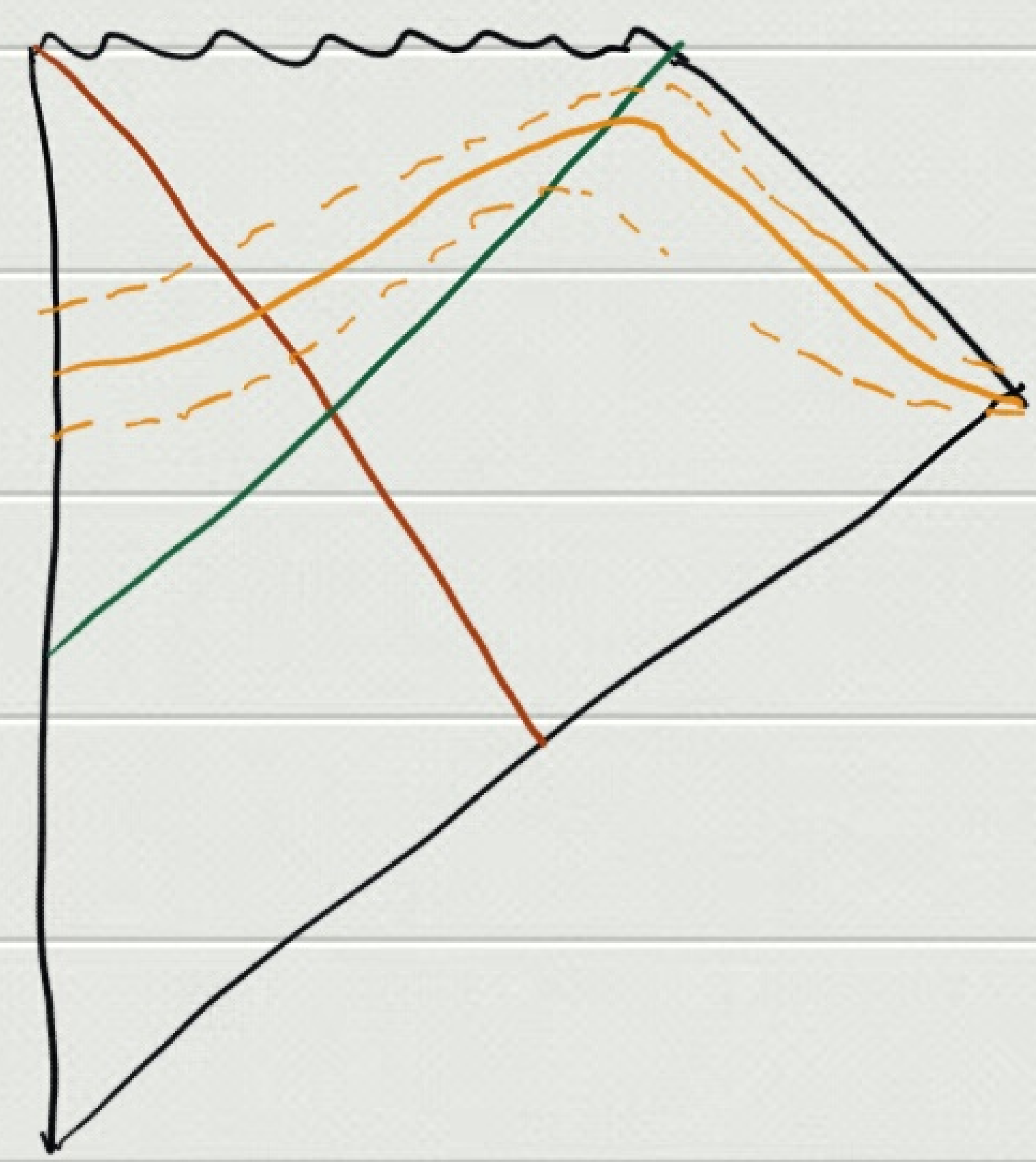
a) we can generate a slice that asymptotes to very large times in the exterior. The only limit is that when the black hole evaporates away completely, curvatures become large. But the black hole remains large (i.e. $r_h \gg l_{pl}$) till almost the very end.

So the nice slices can capture almost all the outgoing Hawking radiation

ii) Notice that as we evolve from one slice to the next the difference in proper time for an observer inside and outside is huge.
We have to cram time of $O(M^3)$ outside into an $O(M)$ proper time inside.

This sometimes causes people to be suspicious of the nice slices but no one has articulated a clear objection based on this.

Now on the nice slices, since no curvature is large, we believe we can do Hamiltonian evolution from one-slice to the next within effective field theory.



So on a sequence of nice slices as shown on the left, we can capture the formation and evaporation of a black hole.

The Cloning Paradox.

Combining the nice-slices and the Page curve leads to the cloning paradox.
The orange slice captures the infalling matter and the outgoing Hawking radiation

So the same information looks like it is present at two spots.

$$|\psi\rangle \rightarrow |\psi\rangle \otimes |\psi\rangle \; ??$$

$\uparrow$

VIOLATION OF UNITARITY!

The difference with the "naive information" paradox is that is unclear how small corrections can resolve this.

On the other hand, note that it is very hard to observe this paradox. and it also involves the infalling observer ← SIGNIFICANCE OF INTERIOR

An observer outside has to

a) collect all the outgoing radiation

b) do a quantum computation to extract information

c) transmit the information to the infalling observe

d) the infalling observer must then do measurements on the infalling matter to observe a violation of Q.M.

One flavour of resolutions suggests that this is impossible / ill-defined ← (Recall Heisenberg uncertainty principle)

Another flavour, which we will not explore much, suggests there is no interior! ← Dramatic violation of effective field theory. but keep an open mind especially today!