QABH - Lecture 25 - 2 Dec 2016

Today, we will discuss a possible resolution of the AMPSS paradoxes through state-dependence.

We will do this by explicitly writing down a construction of the operators $\tilde{a}_w$. These operators will obey all the constraints we demanded provided they are inserted in low point correlators.

More precisely, we construct operator $\phi(x)$ which correctly reproduce Greens functions as computed from WK effective field theory when we consider

$$\langle\psi|\phi(x_1)\ \ldots\ \phi(x_n)|\psi\rangle$$

where

$$n \ll N^2$$

$$|x_i - x_j| \gg \frac{1}{N^2}$$

$$|x_i - x_j| \ll N^2$$

First, let us only consider operators outside the horizon. Then there appears to be no difficulty in constructing the field operators. The modes can be related to modes of single-trace boundary operators. With

$$O_\omega = \frac{1}{T}\int_{-T_c}^{T_c} O(t)\, e^{-i\omega t}\, dt$$

$$a_\omega = O_\omega / \langle [O_\omega, O_\omega^\dagger] \rangle^{1/2}$$

where we have discretized the frequencies. to a finite but possibly large set. eg. $\omega_n \sim n/N^2$.

Now we consider simple polynomials in these modes

$$P = \# a_\omega + \# a_{\omega_1} a_{\omega_2} + \# a_{\omega_1} a_{\omega_2} \ldots a_{\omega_n}$$

we consider the space of such polynomials with total order $\ll N^2$ and total energy in any monomial $\ll N$.

Call this linear space $\mathcal{A}$: this is the set of reasonable experiments that one can perform.

We can choose a basis for $\mathcal{A}$: $A_1 \dots A_D$

Note that the cutoffs we imposed ensure $D \ll e^{N^2}$

We do not include H in the algebra.

As a result, [explain in class], $\forall A_\alpha \in \mathbb{A}$

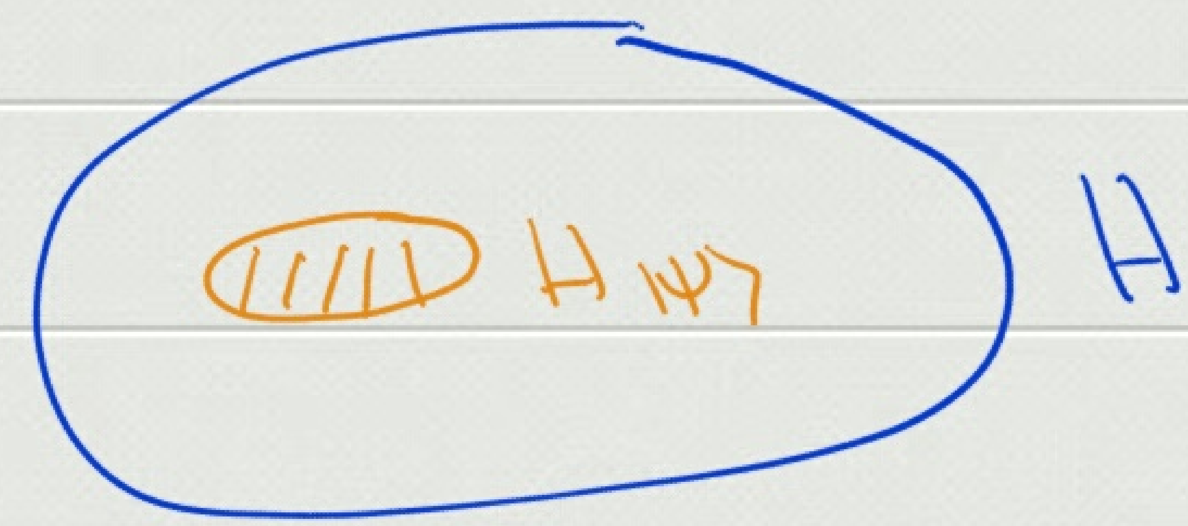
$$A_\alpha |\psi\rangle = 0 \Rightarrow A_\alpha = 0.$$

$|\psi\rangle$ is "separating" w.r.t. $\mathbb{A}$

Next we define a linear space

$$\mathcal{H}_{|\psi\rangle} = \mathbb{A} |\psi\rangle$$

Note $\dim(\mathcal{H}_{|\psi\rangle}) = D$



$H_{|\psi\rangle}$ is a subspace of the full Hilbert space. but it is the only part of the Hilbert space relevant for the infalling observer.

Within effective field theory, all experiments done by the infalling observer move him within $H_{|\psi\rangle}$.

Now we define $\tilde{a}_\omega : \mathcal{H}_{|\psi\rangle} \to \mathcal{H}_{|\psi\rangle}$ through

$$\tilde{a}_\omega A_\alpha |\psi\rangle = A_\alpha e^{-\beta\omega/2} a_\omega^\dagger |\psi\rangle. \quad , \forall A_\alpha$$

Also,

$$\tilde{a}_\omega^\dagger A_\alpha |\psi\rangle = A_\alpha e^{\beta\omega/2} a_\omega |\psi\rangle$$

This is actually a set of linear equations that specifies the action of $\tilde{a}_\omega$ on $\mathcal{H}_{|\psi\rangle}$.

These equations have a solution since, as we argued $A_\alpha |\psi\rangle \neq 0 \quad \forall A_\alpha \neq 0$

We can solve these equations explicitly using our basis $A_1 \ldots A_D$. Set

$$|V_m\rangle = A_m |\psi\rangle$$

$$|u_m\rangle = A_m a_\omega^+ |\psi\rangle e^{-\beta\omega/2}$$

define

$$g_{mn} = \langle V_m | V_n \rangle$$

and

$$\sum_j g^{ij} g_{jk} = \delta^i_k$$

then

$$\tilde{a}_\omega = \sum_{m,n} g^{mn} |u_m\rangle\langle V_n|$$

is an explicit solution for $\tilde{a}_\omega$.

Recall that we had not included H in the algebra.

Separately from the relations above, we can can also impose

$$[H, \tilde{a}_\omega] A_\alpha |\psi\rangle = \omega \tilde{a}_\omega A_\alpha |\psi\rangle$$
$$= \omega A_\alpha a_\omega^\dagger |\psi\rangle e^{-\beta\omega/2}$$

Another way to do this is to consider

$$H_{|\psi\rangle} \oplus H_{H|\psi\rangle} \oplus \ldots H_{H^n|\psi\rangle}$$

On each of these, define

$$\tilde{a}_\omega A_\alpha H^m |\psi\rangle = A_\alpha a_\omega^\dagger H^m |\psi\rangle e^{-\beta\omega/2}$$

We can now explicitly check that this construction obeys the properties we needed:

a) Cross-commutator

$$[\tilde{a}_\omega, a_\omega] A_\alpha |\psi\rangle$$
$$= (\tilde{a}_\omega a_\omega - a_\omega \tilde{a}_\omega) A_\alpha |\psi\rangle$$
$$= (a_\omega A_\alpha a_\omega^\dagger - a_\omega A_\alpha a_\omega^\dagger) |\psi\rangle e^{-\beta\omega/2}$$
$$= 0.$$

v) canonical commutator

$$[\tilde{a}_\omega, \tilde{a}_\omega^+] A_\alpha |\psi\rangle$$

$$= (\tilde{a}_\omega \tilde{a}_\omega^+ - \tilde{a}_\omega^+ \tilde{a}_\omega) A_\alpha |\psi\rangle$$

$$= \tilde{a}_\omega A_\alpha a_\omega |\psi\rangle e^{\beta\omega/2} - \tilde{a}_\omega^+ A_\alpha a_\omega^+ |\psi\rangle e^{-\beta\omega/2}$$

$$= A_\alpha a_\omega a_\omega^+ |\psi\rangle - A_\alpha a_\omega^+ a_\omega |\psi\rangle.$$

$$= A_\alpha |\psi\rangle$$

Note we do not have (as an operator relation)

$$[\tilde{a}_\omega, \tilde{a}_\omega^+] = 1$$

or even

$$[\tilde{a}_\omega, a_\omega] = 0$$

rather these relations are true inside low-point correlators.

Let us check the two-point functions

$$\langle \psi | \tilde{a}_\omega \tilde{a}_\omega^\dagger | \psi \rangle = \langle \psi | \tilde{a}_\omega a_\omega | \psi \rangle e^{\beta\omega/2}$$

$$= \langle \psi | a_\omega a_\omega^\dagger | \psi \rangle$$

$$= \frac{1}{1-e^{-\beta\omega}}$$

Also $\langle \psi | \tilde{a}_\omega a_\omega | \psi \rangle = \langle \psi | a_\omega a_\omega^\dagger | \psi \rangle e^{-\beta\omega/2}$

$$= \frac{e^{-\beta\omega/2}}{1-e^{-\beta\omega}}.$$

We can also evaluate N_a. Recall that

With $\alpha_\omega = a_\omega - e^{-\beta\omega/2}\tilde{a}^\dagger_\omega$

$$\beta_\omega = \tilde{a}_\omega - e^{-\beta\omega/2} a^\dagger_\omega$$

$$N_a = \alpha^\dagger_\omega \alpha_\omega + \beta^\dagger_\omega \beta_\omega .$$

But $\alpha_\omega |\psi\rangle = 0 = \beta_\omega |\psi\rangle$

so

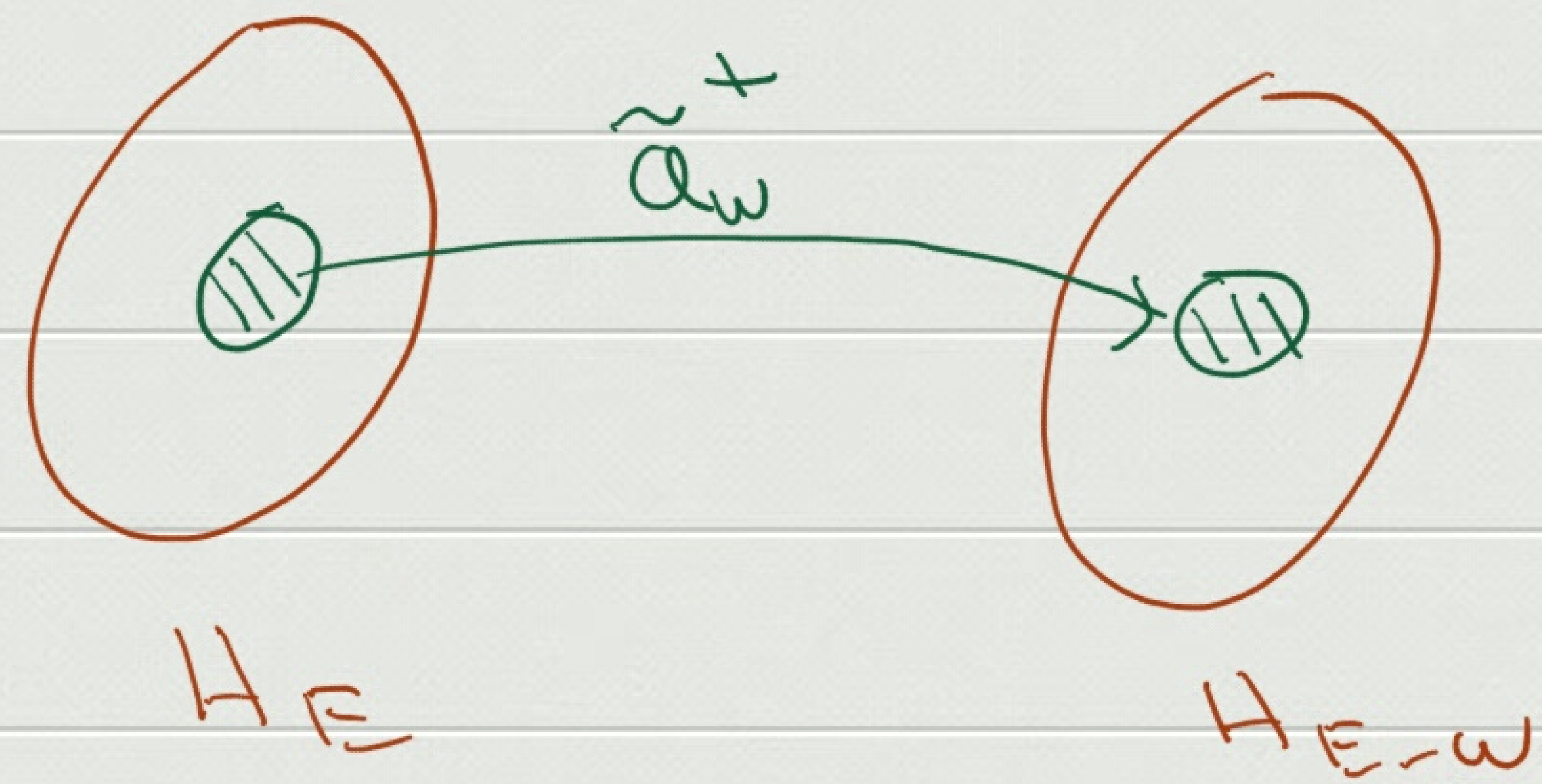
$$\langle\psi| N_a |\psi\rangle = 0!$$

State-dependence

As defined, the operators $\tilde{a}_\omega$ are state-dependent: they depend on the state $|\Psi\rangle$, but this dependence cannot be detected by an infalling observer.

For completeness, let us see how the various paradoxes are avoided.

resolution of the lack of a left-inverse paradox



Here $\tilde{a}^+_\omega$ act only in a small subspace $H_{|\psi\rangle} \cap H_E$ and maps this to another subspace $H_{|\psi\rangle} \cap H_{E-\omega}$.
The fact that $\dim(H_{E-\omega}) < \dim(H_E)$ is irrelevant here. $[\tilde{a}_\omega, \tilde{a}^+_\omega] \doteq 1$ is true only inside correlators and not as an operator relation.

Resolution to the occupancy argument

$$Z \langle\psi| \tilde{a}_\omega^{\psi} \tilde{a}_\omega^{+\psi} |\psi\rangle \neq \mathrm{Tr}\,(\tilde{a}_\omega^{\psi} \tilde{a}_\omega^{+\psi}).$$

We might try

$$Z \langle\psi| \tilde{a}_\omega^{\psi} \tilde{a}_\omega^{+,\psi} |\psi\rangle = \sum_E \langle E| \tilde{a}_\omega^{|E\rangle} \tilde{a}_\omega^{+,|E\rangle} e^{-\beta E} |E\rangle$$

but now we lose cyclicity of the trace due to state-dependence.

Resolution to the $N_a \neq 0$ argument

It is true that $\langle E | N_a | E \rangle = 0 + O(\frac{1}{N})$

and $\langle N_i | N_a | N_i \rangle = 0$

But for state dependent operators

$$\sum_E \langle E | N_a^E | E \rangle \neq \sum_{N_i} \langle N_i | N_a^{N_i} | N_i \rangle$$

Change of basis is impermissible!

Survey

The state-dependent construction of the $\tilde{a}_w$ does not prove the absence of firewalls. There seem to be two alternatives

1) Only black holes formed from collapse have regular interiors. In that case we can construct state-independent operators we simply taking $\sum_i \tilde{a}_w^{|\psi_i\rangle}$, where $|\psi_i\rangle$ runs over smooth black holes

2) All black holes have smooth interiors, but interior operators are state-dependent

3) AdS/CFT is incomplete and does not describe B.H. interiors.