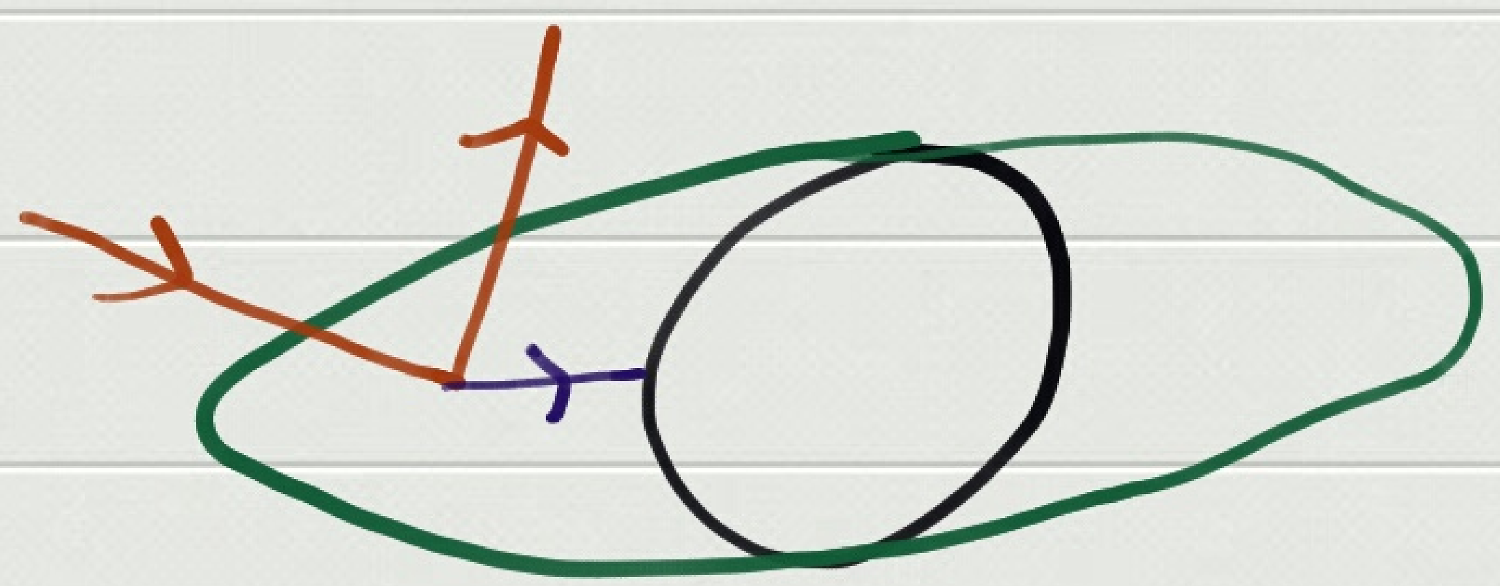
SERC SCHOOL, LECTURE 4, 30 NOV 2015

Last time, we discussed the Penrose process



In this a particle enters the ergosphere, ejects another particle and exits. We found that

$$\delta L \le \frac{\delta M}{\Omega}$$

While M can decrease, it turns out that the area cannot. To see this, we write the area of the horizon.

The area is defined as the induced area of the intersection of the horizon with a Cauchy surface.

We find (tutorial) that

$$A = \int \sqrt{g_{\theta\theta}\, g_{\phi\phi}}\, d\theta\, d\phi = \int d\theta\, d\phi\, (r_+^2 + a^2) \sin\theta$$
$$= 4\pi\, (r_+^2 + a^2).$$

Converting into M, J, q we use the fact that

$$r_+^2 + \frac{J^2}{M^2} + e^2 - 2Mr_+ = 0$$

$$\Rightarrow \quad r_+ = M + \sqrt{M^2 - \left(\frac{J^2}{M^2} + e^2\right)}$$

and $r_+^2 + a^2 = 2Mr_+ - e^2$

$$= 2M^2 + 2\sqrt{M^4 - (J^2 + e^2M^2)} - e^2$$

We have

$$\frac{\partial A}{\partial M} = \left(4M + \frac{4M^3 - 2e^2M}{\sqrt{M^4 - (J^2 + e^2M^2)}}\right)4\pi$$

$$= 8\pi\left(2M\sqrt{M^2-(a^2+e^2)} + 2M^2-e^2\right)/\sqrt{M^2-(a^2+e^2)}$$

$$= 8\pi\,(r_+^2+a^2)/\sqrt{M^2-(a^2+e^2)}$$

We define

$$k = \frac{\sqrt{M^2-(a^2+e^2)}}{2M\sqrt{M^2-(a^2+e^2)}+2M^2-e^2}$$

Second we also have,

$$\left(\frac{\partial A}{\partial J}\right)_M = -\frac{8\pi J}{\sqrt{M^4-(J^2+e^2M^2)}} = -\frac{8\pi a}{\sqrt{M^2-(a^2+e^2)}}$$

Now notice that

$$\left(\frac{\partial A}{\partial J}\right)_M \Big/ \left(\frac{\partial A}{\partial M}\right)_J = \frac{-a}{r_+^2 + a^2} = -\Omega$$

therefore we have (in the Penrose process)

$$\delta A = \frac{8\pi}{\kappa} \left(\delta M - \Omega \delta J \right)$$

We showed earlier that $\delta M - \Omega \delta J > 0$
and so, we see

$\delta A \geq 0 \rightarrow$ Second law of B.H. thermodynamics

Second

$$\left(\frac{\partial M}{\partial J}\right)_A = -\left(\frac{\partial A}{\partial J}\right)_M \Big/ \left(\frac{\partial A}{\partial M}\right)_J$$

$$= \frac{a}{r_+^2 + a^2} = \Omega$$

So we can write

$$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega \, \delta J$$

First law of B.H. thermodynamics.

You might feel that we derived these laws for a very special case, but all that we used was the fact that the particle falling into the B.H. had positive energy w.r.t. $k + \Omega R$.
This is what goes into the general proof of the 2nd law as well.

At this point, I would like to pause and give you an overview of the remaing 5 lectures.

So far we have found the following:

a) a set of B.H. solutions that have the property of having smooth horizons.

b). that have no hair

c) that behave like ordinary thermodynamic objects.

Hawking found (as we will review) that when quantum effects are added, the analogy with thermodynamics becomes precise

More specifically black holes emit black body radiation with a temperature $T = \frac{k}{2\pi}$.

This suggests they have an entropy

$$S_{bh} = \frac{A}{4}.$$

But more interestingly, this means that they radiate (using the Stefan Boltzmann Law)

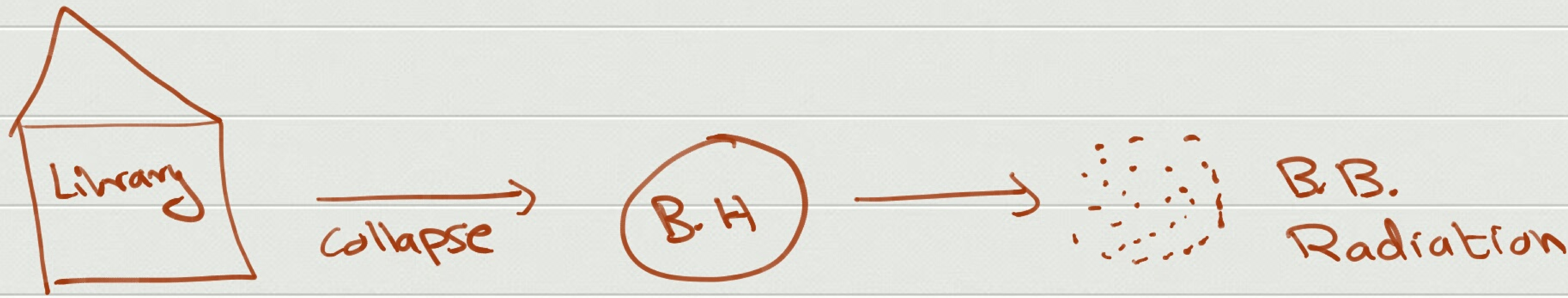
$$\frac{dM}{dt} = -\sigma T^4 A$$

So, for a Schwarzschild black hole, $T \sim \frac{1}{M}$, $A \sim M^2$ and so

$$\frac{dM}{dt} \propto -\frac{1}{M^2}$$

$$\Rightarrow T_{life} \propto M^3$$

Now Hawking posed the following paradox. If B.H. radiation is perfect black body radiation, the we seem to have the following process.



This suggests a loss of information. Multiple initial states seem to lead to the same final state. Recall, a very fundamental property of both classical & quantum mechanics is that evolution is reversible.

The Schrodinger equation also has the property that

$$i \frac{\partial |\psi\rangle}{\partial t} = H |\psi\rangle.$$

If this picture was really correct, then we would have a violation of this basic principle.

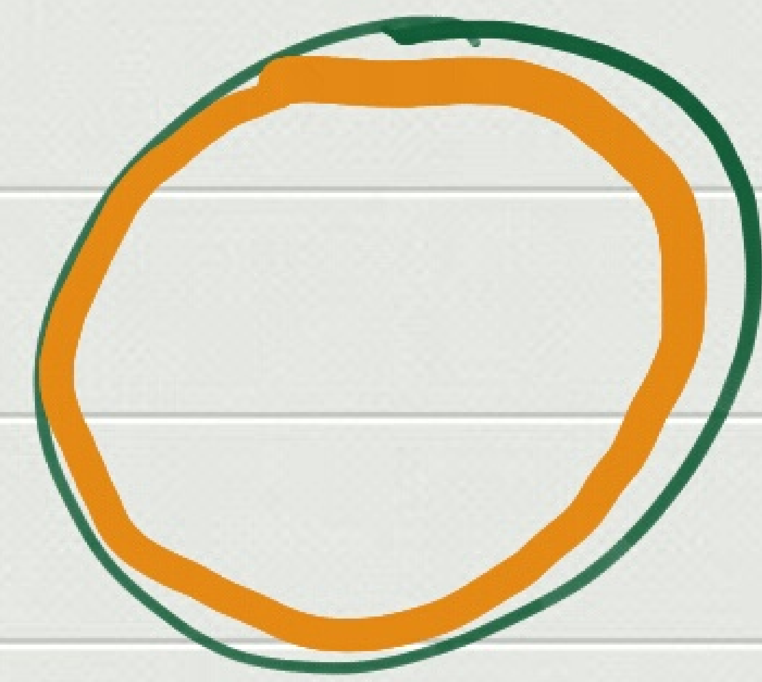
If black holes had hair, we could resolve this by appealing to the fact that Hawking radiation depends on these other parameters, and therefore can preserve information.

This formulation is the old information paradox:

The resolution to this paradox is as follows. Hawking's calculation tells us that some properties of the B.H. radiation are thermal. It does not tell us the radiation is exactly thermal. Because the final phase space is so large, information can be stored in tiny correlations between different emitted quanta. Hawking's calculation is not precise enough to lead to a paradox.

Recently, various people tried to show that such small corrections can't fix the information paradox.
These arguments are of two types. One tries to use the entanglement entropy of different regions of space and set up a paradox at each step during Hawking radiation.
The second tries to use general properties of correlators behind the horizon to argue that these field operators cannot exist in a unitary theory.

One possible resolution is firewalls



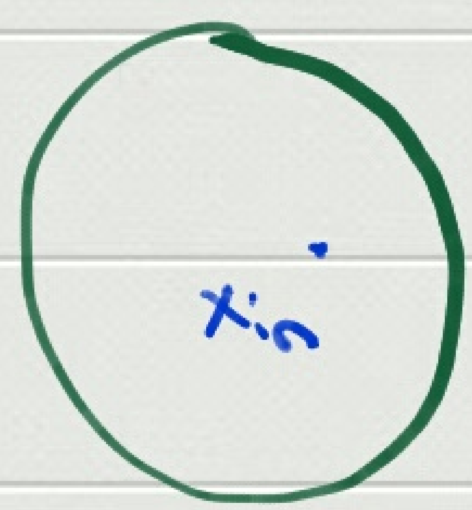
If true, this would be a dramatic violation of EFT

We expect Q.G. effects to be important when curvature is large, not near horizons of big black holes.

[Explain using effective action]

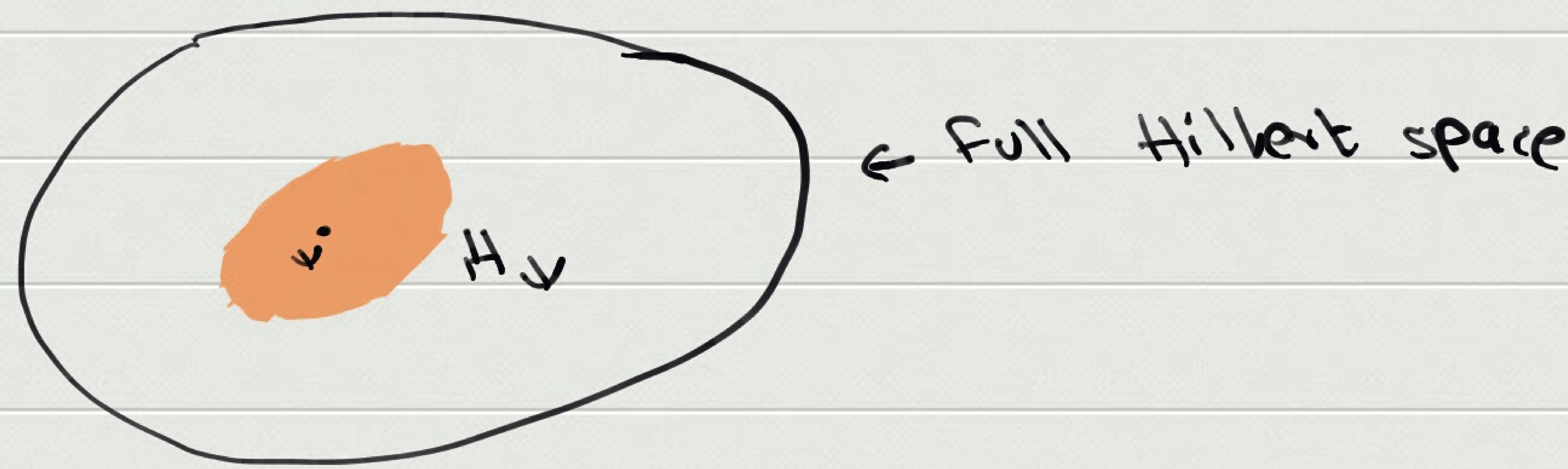
$$S = \frac{1}{16\pi G}\int\left(\sqrt{-g}\,R + \sqrt{-g}\,\frac{R^2}{M_{Pl}^2} + \ldots\right)$$

The other solution is to accept two limitations on the properties of local operators in Q.G.
One is that locality is not exact in quantum gravity. So in a sense the dof inside the B.H. are scrambled versions of the dof outside the B.H.



the operator at x_{in} can be written as a scrambled version of the dof outside.

The second idea is state-dependence. Local operators are defined in a given state and in its neighbourhood!



Explain significance of this in terms of ordinary observables.

As we will see, if we accept these, then we can resolve the modern info par.

More on this in the last lecture.