Lecture 8, SERC SCHOOL

[First, we complete the derivation of Hawking rad. See lecture 7 notes]

We have now derived the formula for Hawking radiation in several ways. The net effect is that the b_ω modes (outgoing modes near the horizon) are thermally populated.

So there is a net outgoing flux of energy

$$\sigma T^4 A = \frac{\sigma \quad 4\pi \cdot (4M^2)}{(8\pi)^4 \; M^4} = \frac{\sigma}{32\pi^3 M^2}$$

So far in our discussion of QFT in curved space, we have not considered the back-reaction of fields on the geometry.
If we do consider this then we find that by conservation of energy the mass of the black hole decreases.

The crudest version of the information paradox is as follows: start with matter in a pure state. Let it collapse and let the B.H. evaporate. So it looks like we have pure state $\longrightarrow$ mixed state.

This is not possible under any unitary evolution.
If

$$p^2 = p$$

then

$$(U p U^\dagger)^2 = U p U^\dagger$$

Answer

The answer to this is simple: we have just found that 2-pt functions $\langle b_\omega b_\omega^\dagger \rangle$ are thermal.
This does not imply that the final state is thermal.

More precisely we may have $\rho_1^2 = \rho_1$; $\rho_2^2 \neq \rho_2$
and still have

$$\mathrm{tr}(\rho_1 A_\alpha) = \mathrm{tr}(\rho_2 A_\alpha) + e^{-S/2}$$

for a large class of observables.

<u>Example:</u>

Consider some stat. mech. system with energy eigenstates $|E_i\rangle$

Observables that thermalize obey the eigenstate thermalization hypothesis.

See papers by Srednicki for the ETH

$$\langle E_i | A_\alpha | E_j \rangle = A_\alpha(E)\,\delta_{ij} + e^{-S/2} R_{ij}$$

where S is the density of states at $(E_i + E_j)/2$.

Now consider the state

$$|\psi\rangle = \sum_E \frac{e^{-\beta E/2}}{\sqrt{Z}} |E\rangle$$

$$\langle \psi | A_\alpha | \psi \rangle = \sum_E A_\alpha(E) \frac{e^{-\beta E}}{Z} + \sum_{i,j} e^{-S/2} \frac{e^{-\beta(E_i+E_j)/2}}{Z} R_{ij}$$

At a given temperature β, only a range of energies is relevant around some E_0. So the sum over i,j runs of e^{2S} terms. But these terms contribute randomly. So the sum is $O(e^S)$.

So we get

$$e^{-S/2} \frac{e^{-\beta E+S}}{Z} = e^{-S/2} \frac{e^{-\beta F}}{e^{-\beta F}} = e^{-S/2}.$$

So we have proved

$$\langle \psi | A_\alpha | \psi \rangle = \frac{1}{Z} \operatorname{tr} \left(e^{-\beta H} A_\alpha \right) + e^{-S/2}$$

as advertised.

In fact generic states $|\psi\rangle$ behave thermally, not just the one in the specific example above.

So the fact that simple correlators behave thermally is far from sufficient to conclude that the state is mixed.

Some people try to compute corrections to Hawking radiation to resolve the paradox. This is futile! We see that unless one has non-perturbative control, there is no paradox

Hawking's calculation is not precise enough to lead to a paradox.

Moreover, non-perturbative gravity is especially difficult because we lose notions of locality.

To define gravity non-perturbatively, we need to consider the path integral

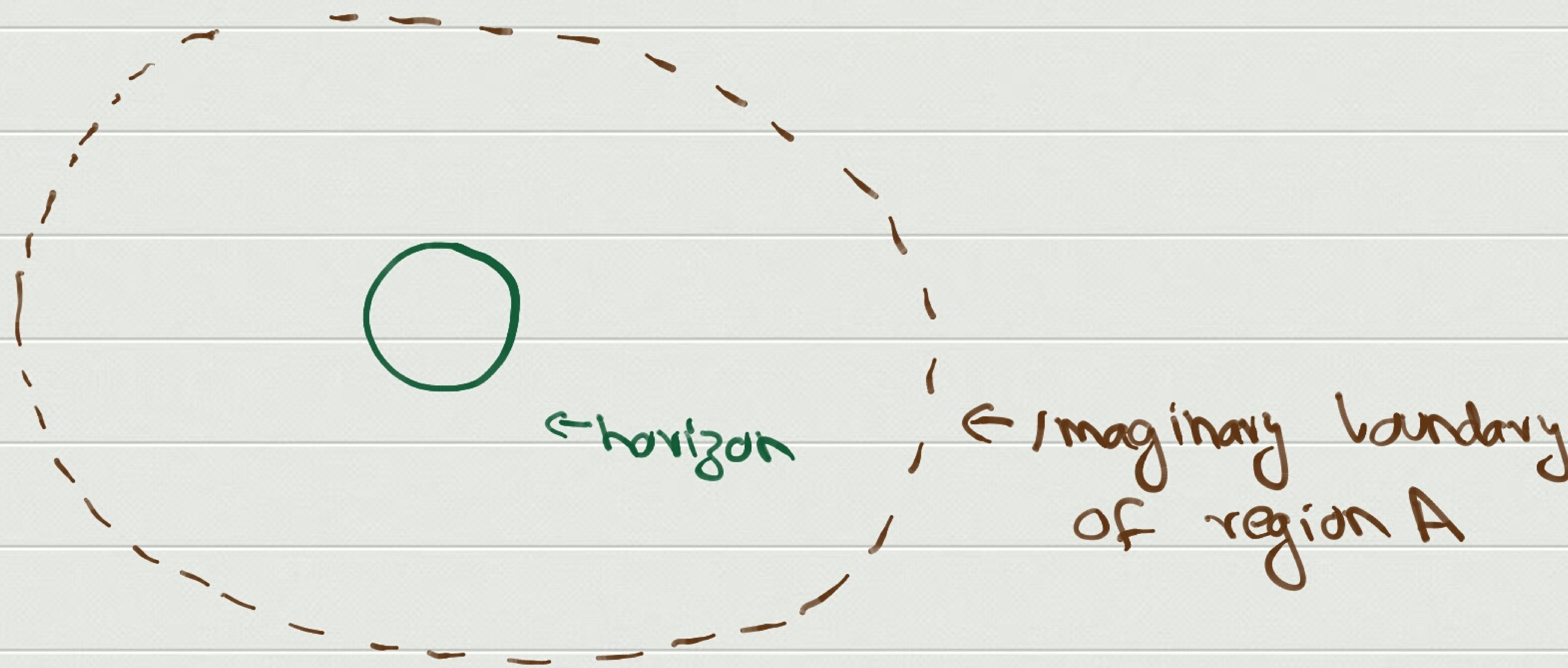
$$\int e^{-S} \, Dg_{\mu\nu}$$

where S is the action now, not the entropy!

But locality can only be defined about a saddle-pt and is therefore only perturbative! It requires a background metric $g_{\mu\nu}$!

Page curve

We can actually demand something more detailed than simply the fact that the final state is pure.



Consider a region A "very far" from the B.H. and define

$$\rho_A = \underset{\text{complement of } A}{\mathrm{tr}} \left(|\psi\rangle\langle\psi| \right)$$

and consider

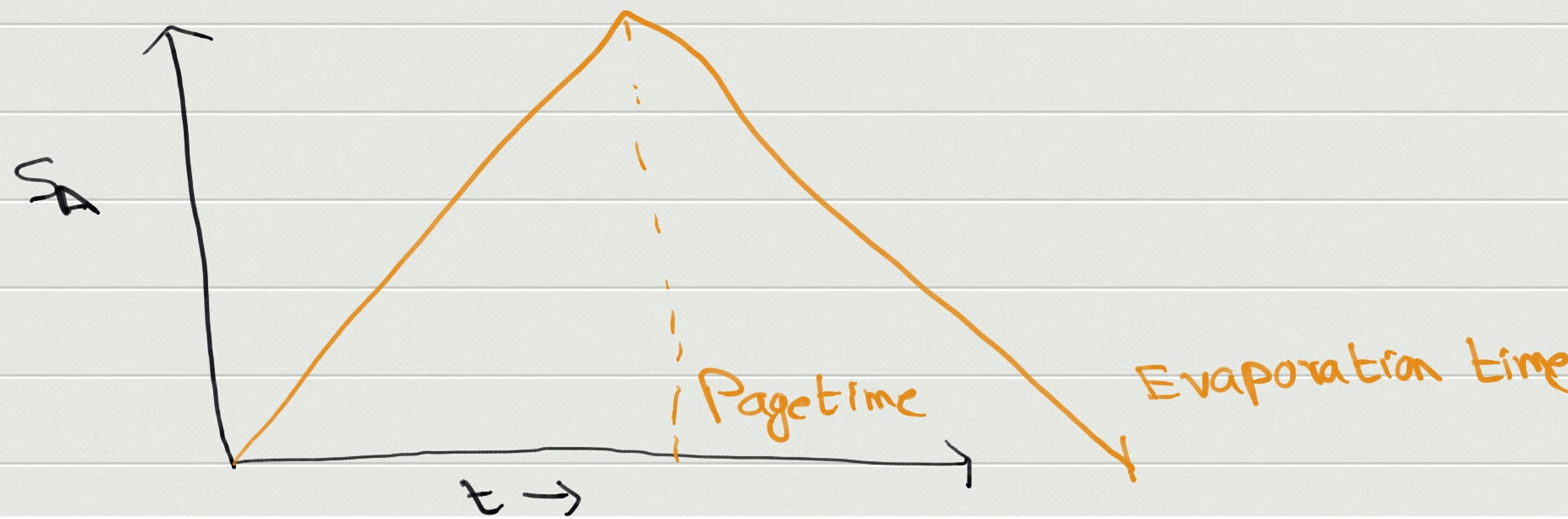
$$S_A = -\mathrm{tr}\left(\rho_A \ln \rho_A\right) - S_0$$

called the von Neumann entropy where S_0 is the von Neumann entropy of the vacuum (non-zero; recall the vacuum is entangled.)

At $t=0$, S_A is 0. [region A is unpopulated with any energy]

As Hawking radiation begins to reach A, S_A increases.

Now Page showed that for a generic state



Roughly the proof goes as follows. Consider a system S with

$$\mathcal{H}_S = \mathcal{H}_A \otimes \mathcal{H}_{\tilde{A}}$$

full Hilbert space

Hilbert space of A

Hilbert space of complement of A

and also

$$E_S = E_A + E_{\tilde{A}}$$

then as we redistribute the total energy in a typical state of S, we get the curve shown previously.