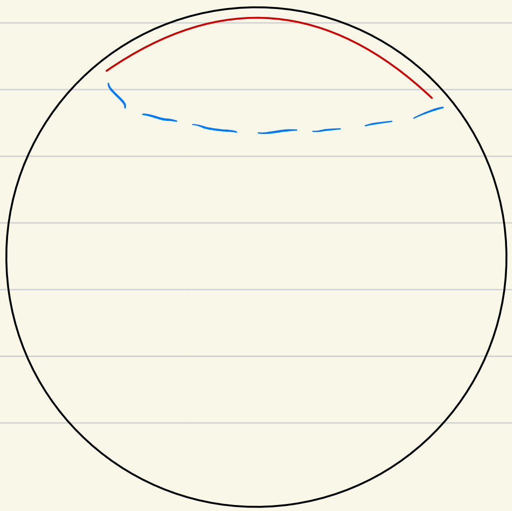
18 March

Lecture 18 : Introduction to Islands

First, we complete the discussion of the nontrivial E.W.

It is also relevant to ask if the arguments used to establish the principle of holography of information can be used to understand this

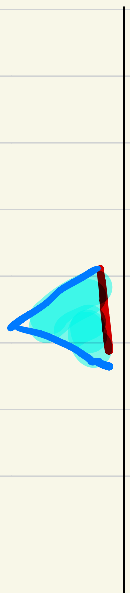
This principle gives us a **bulk** argument for why the entire boundary knows about the entire bulk



These arguments can also plausibly be extended to argue for the simple subregion dualities. [See prev. figure]

First, lets see how gravity is important.

Even without gravity,
it is not surprising that the boundary causal diamond knows about the bulk causal wedge

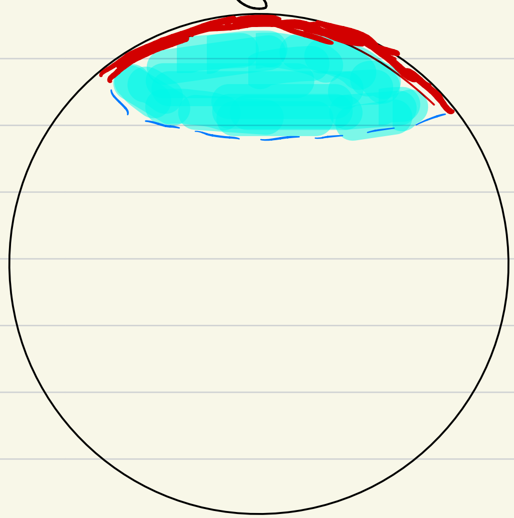


← mapping between red region and blue wedge works even without gravity

But subregion duality states the boundary region R has information about the bulk causal wedge.

What is surprising from a bulk perspective:

"why should the red region have information about the blue region?"



Asked another way.

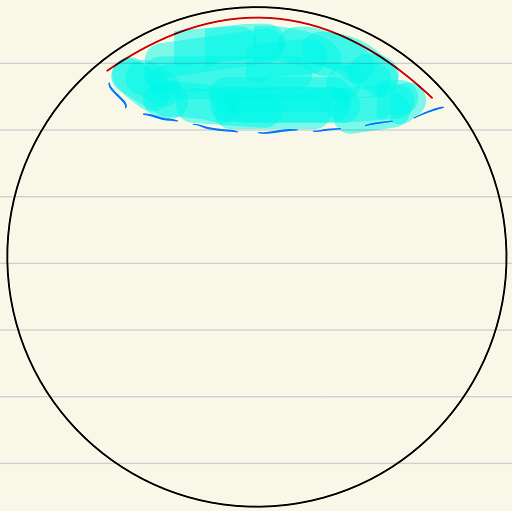
From a **bulk perspective**, given the region

why can we complete the diamond purely on the boundary.

why can't "new information" come in from the bulk?

From AdS/CFT it seems obvious, but otherwise it is puzzling.

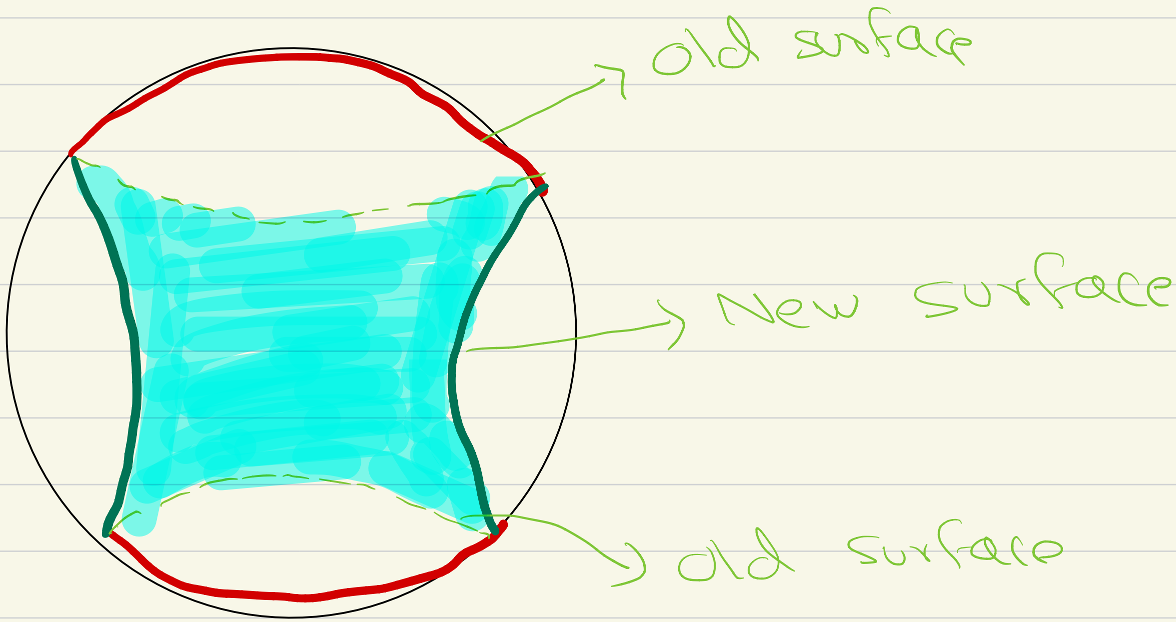
What we need to show is that



the information in the red region is sufficient to obtain information in the boundary causal diamond built on the red region.

This relies on the Hamiltonian being a boundary term.

But this picture seems to require some new insight to understand. From a bulk perspective



Replica trick.

A powerful technique used to analyze entanglement entropy is the replica trick.

The idea is as follows.

First, say that we want to obtain an expression for the ground-state wave-functional of quantum fields.

$$\psi[\phi(x)]$$

For every field configuration, this gives a number.

One representation is as follows

$$\psi[\phi(x)] \propto \int_{\phi(x,-\infty)=0}^{\phi(x,0)=\phi(x)} D\phi \; e^{-S_E[\phi]}$$

where the integral is done in Euclidean space over a half-space

remember that the Euclidean path integral evaluates

$$\langle \phi_1(x) | \, e^{-H\tau} \, | \phi_2(x) \rangle$$

when performed over Euclidean time τ

This is $\sum_E \langle \phi_1(x) | E \rangle \langle E | \phi_2(x) \rangle e^{-E\tau}$

For long times the vacuum state dominates

When viewed as a functional of $\phi_1(x)$ this tells us

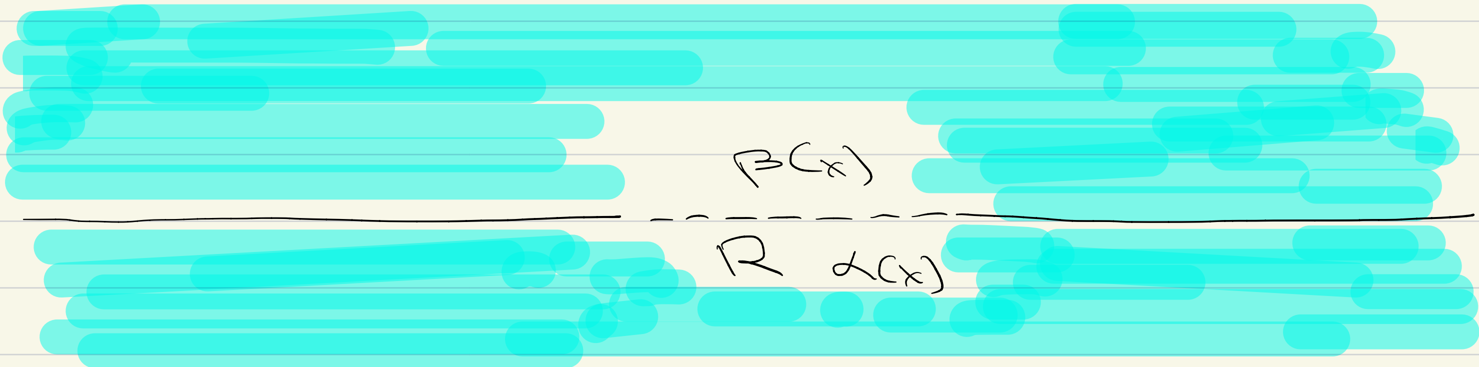
$$\langle \phi_1(x) | \Omega \rangle$$

up to a normalization.

Now say we want to compute a density matrix of a region R when the global state is the vacuum.

This can be done through

$$P(\alpha, \beta) \propto \int_{\substack{\phi(x,0^-)=\alpha(x) \\ x \in R}}^{\phi(x,0^+)=\beta(x),\ x\in R} e^{-S}\, D\phi$$



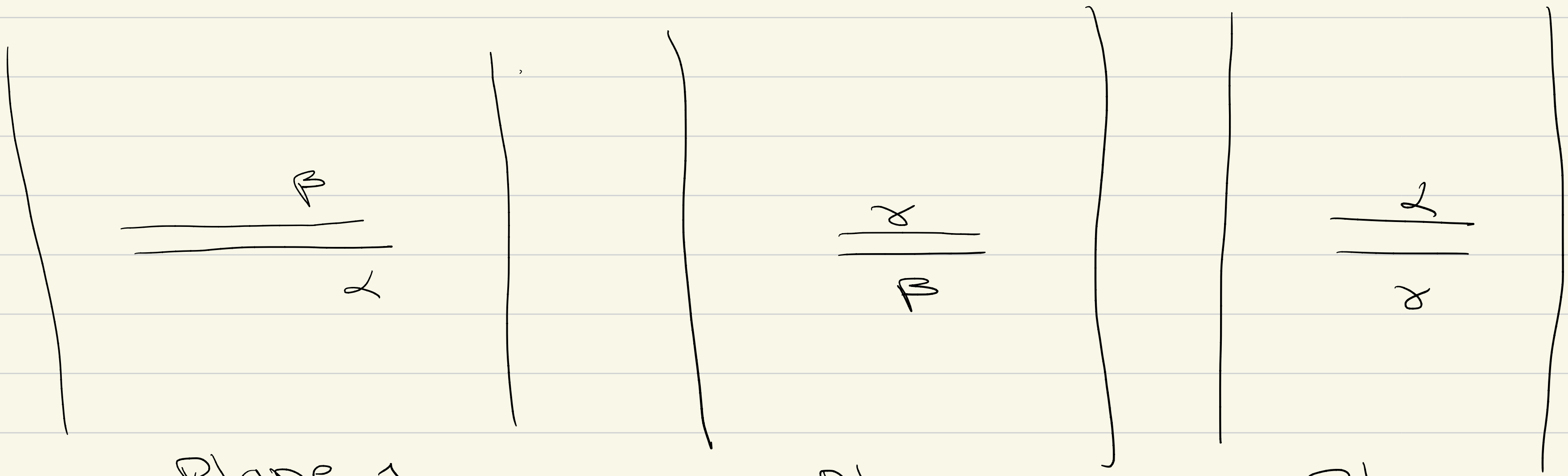
Integral over shaded region with a cut in the middle.

Now say that we want to compute

$$\mathrm{tr}(\rho^n)$$

Then we take **n copies** of the plane.

We sew the upper cut on plane j with the lower cut on plane $j+1$. and the upper cut on plane n with lower cut on plane 1



Plane 1 Plane 2 Plane 3

So

$$\mathrm{tr}\,\rho^n = \frac{Z(n)}{Z(1)^n}$$

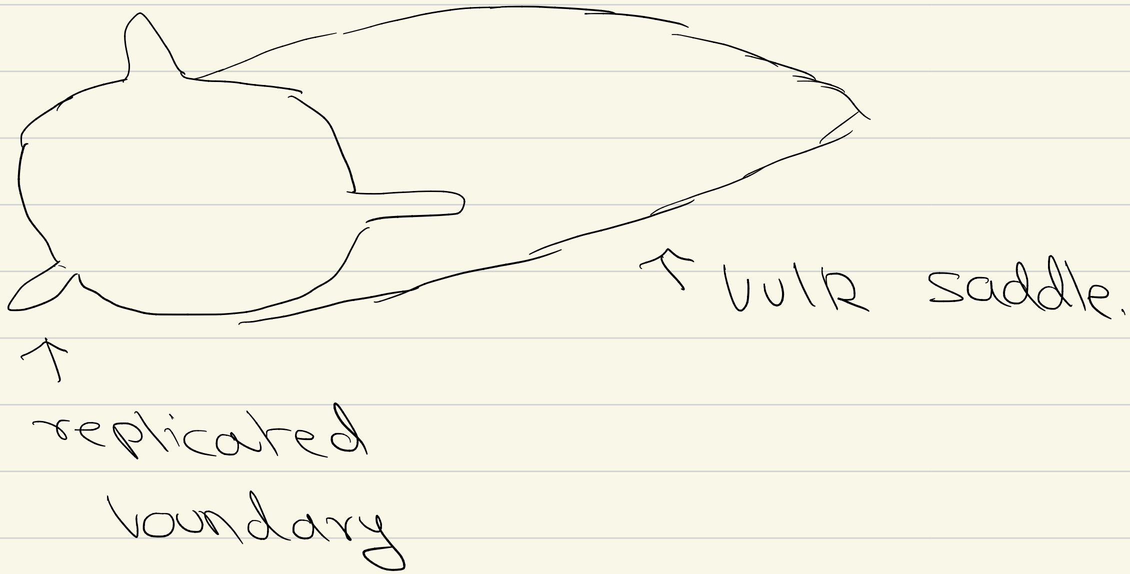
To compute the von Neumann entropy we use

$$-\mathrm{tr}(\rho \log \rho) = \lim_{n \to 1} \frac{1}{1-n} \log \mathrm{tr}\,\rho^n$$

To obtain the limit, we differentiate both num and denominator w.r.t. n

$$S = -\partial_n \left[\log Z(n) - n \log Z(1) \right] \Big|_{n=1}$$

In holographic theories, the idea is to find a **bulk saddle** for the replicated voundary manifold



If we find the bulk metric, the bulk action gives us an approximation to the voundary path-integral.

Now the main point is as follows.

1) For $n=1$, bulk metric is AdS

2) We need the metric "near" $n=1$.

3) We analytically continue the metric "away" from $n=1$, by thinking of a metric which has small conical singularities proportional to $(n-1)$

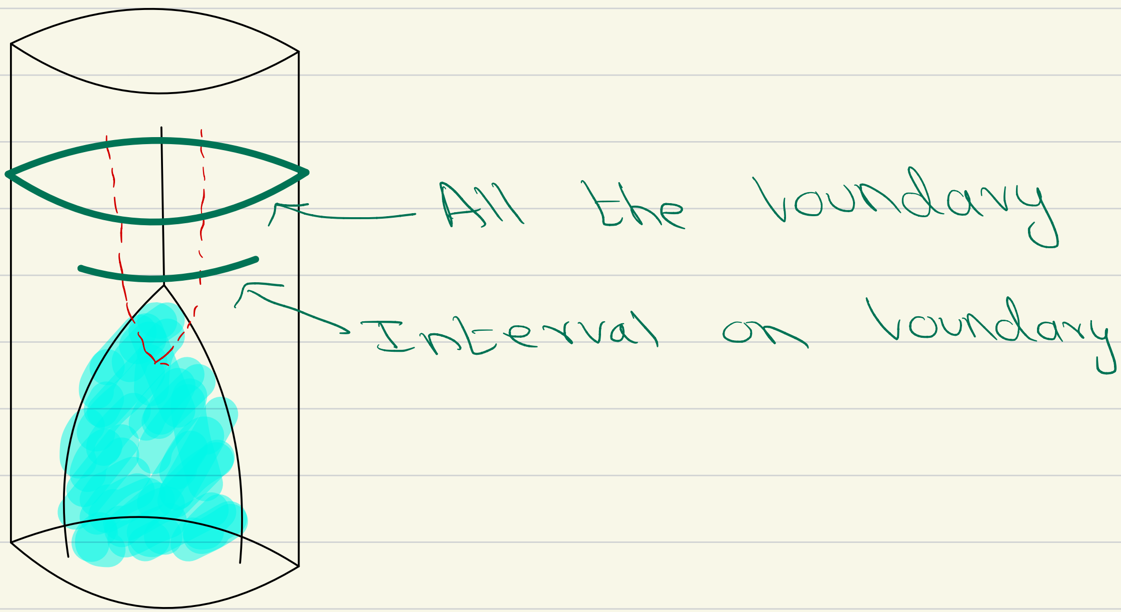
4) Computing the bulk action for this metric and taking the limit yields the area law (and also leading quantum corrections)

5) Perhaps if we had not known the answer ahead of time, we might not have guessed the right analytic continuation.

CFTs coupled to baths

So far we have discussed the entropy of intervals of the CFT

We could consider a CFT state dual to a black hole in the bulk



In this state, we could ask about

1) Entanglement entropy of some interval
2) Entropy of the entire boundary.

If we ask about 2 **(and if the b.h. formed from collapse)** we expect the

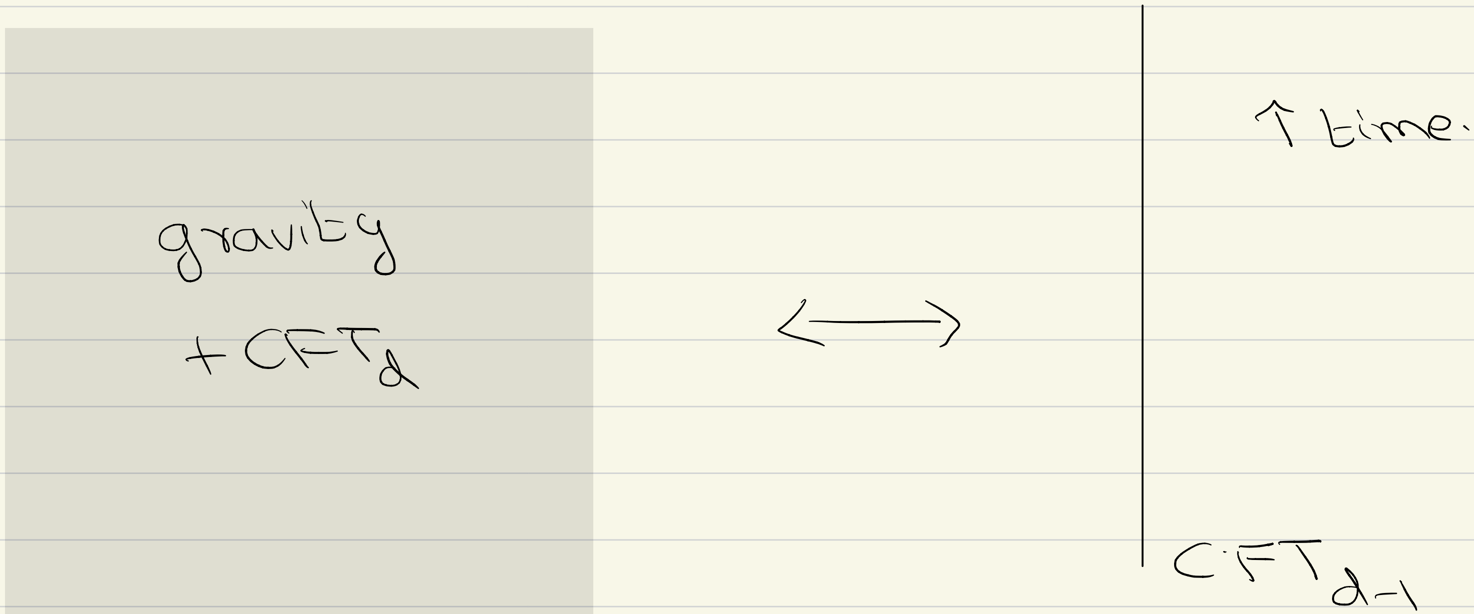
"entropy of the entire state always remains 0."

This is <u>consistent</u> with the principle of holography of information.

This is <u>also</u> <u>true</u> if the black hole is a small black hole and evaporates.

But we can ask a different question [Recap from Lecture 16, last part]

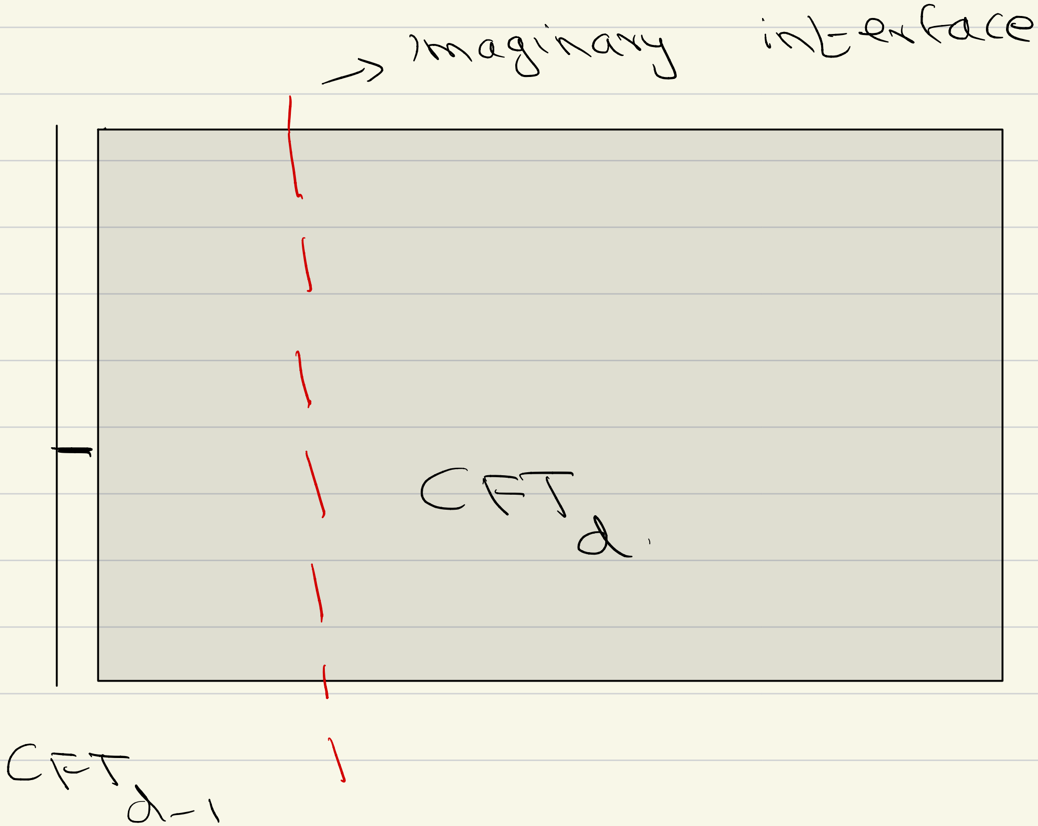
Consider the full CFT. we will call it CFT_{d-1}



This has a gravity dual. we take the matter sector of this dual to be CFT_d

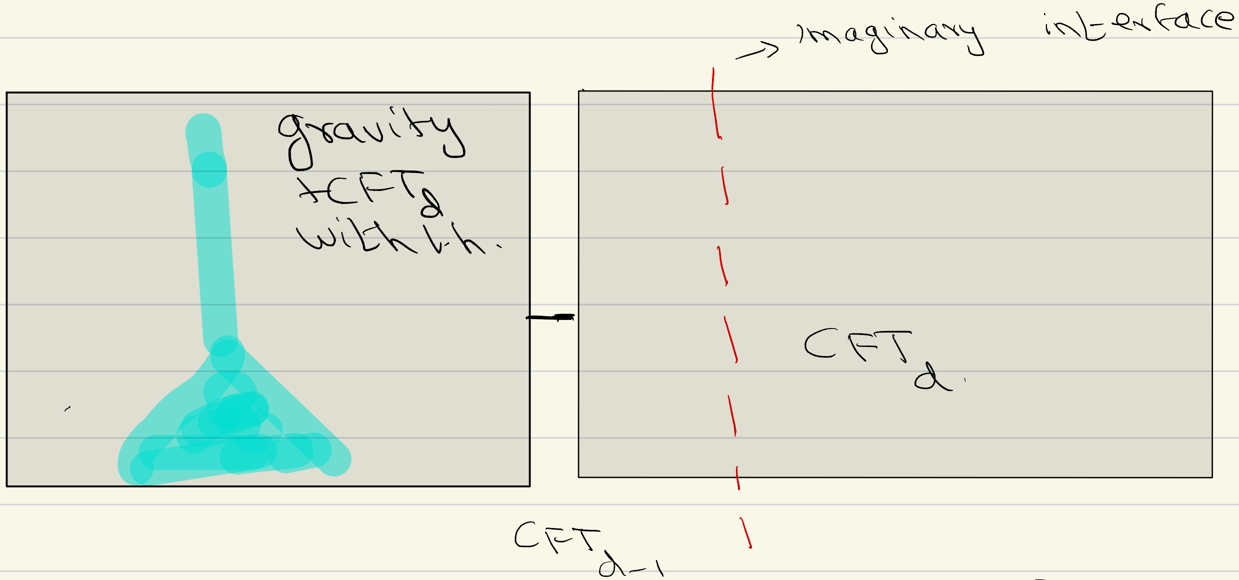
We now couple the CFT_{d-1} to CFT_d in flat space.

The CFT_d may or may not be holographic [later we will consider the case where it is.]



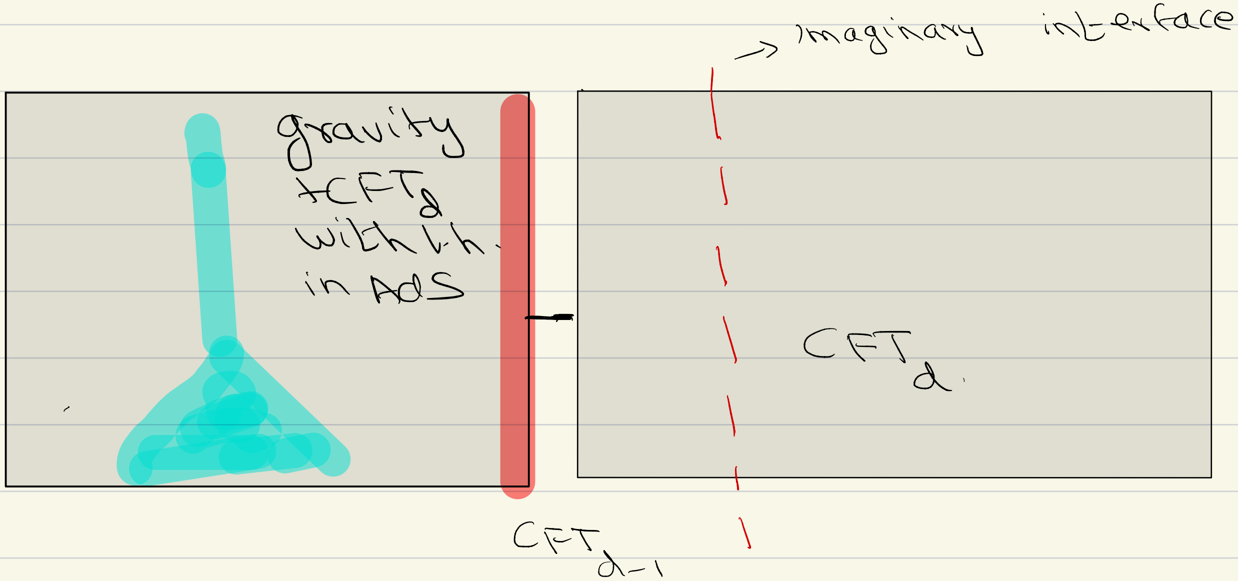
We now seek to compute the entanglement entropy across an imaginary surface in this **nongravitational system**.

Now we can draw another picture for this process.



This has led to the use of words that this is "computing how information "emerges" from black holes."

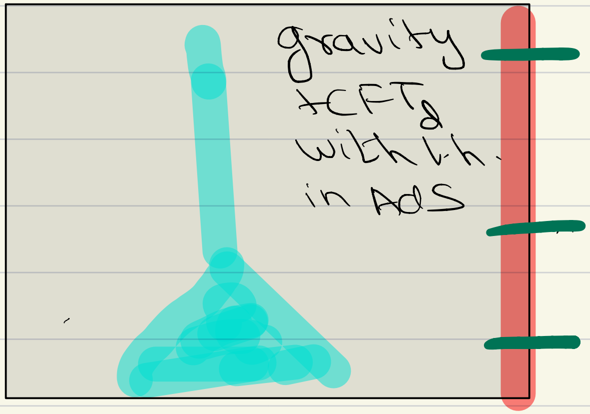
But these words are somewhat sloppy.



Holography of information $\Longrightarrow$ information is already present in the red region before the coupling is turned on.

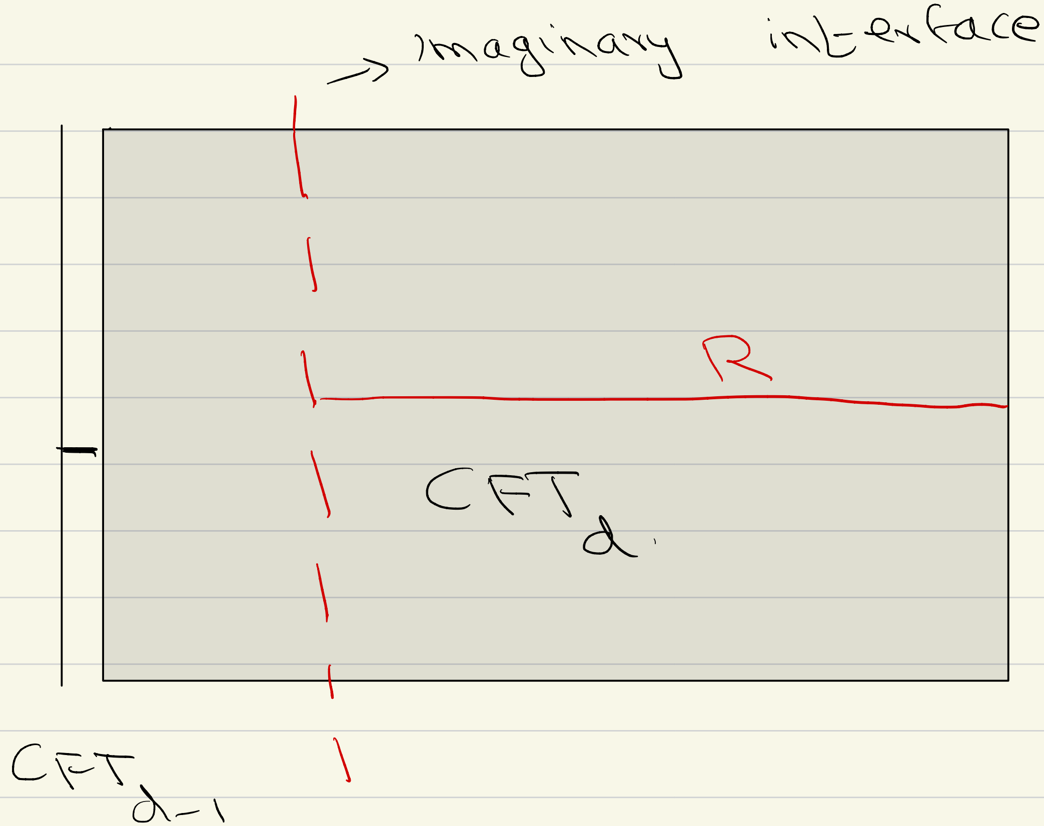
So it is more accurate to not use these words and simply consider the nongravitational question, which is already interesting.

Note also that this does not tell us about unitarity of bh evaporation



entropy at any cut is 0, which already shows unitarity.

But this is already clear from the fact that the red region always has entropy 0.



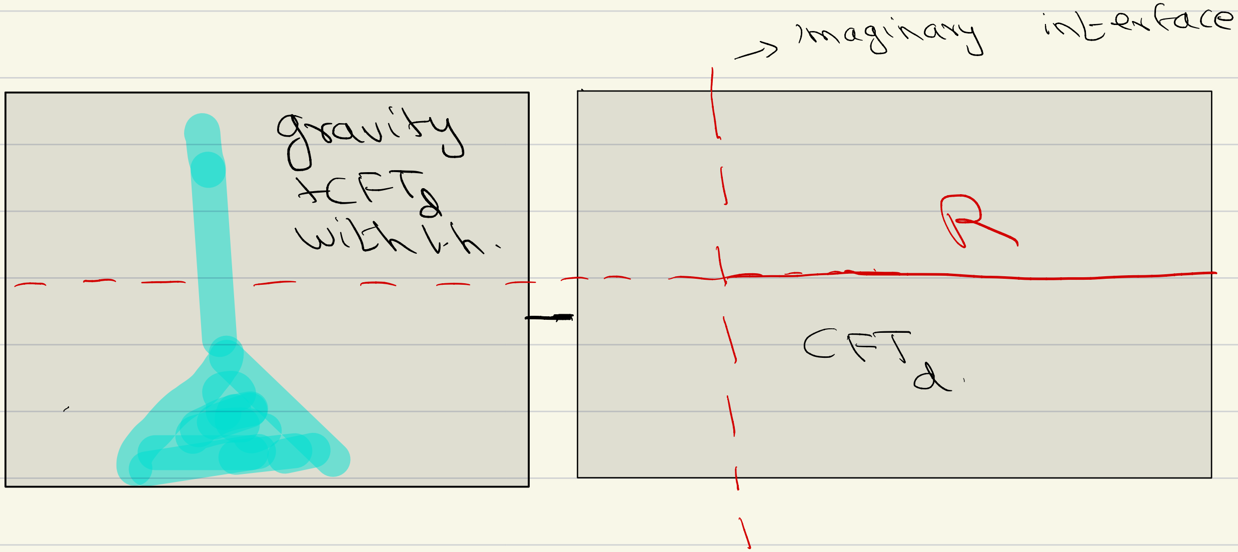
More interesting question is:

"Given CFT_{d-1} has a gravitational dual can we generalize AdS/CFT formulas to compute the entropy of R."

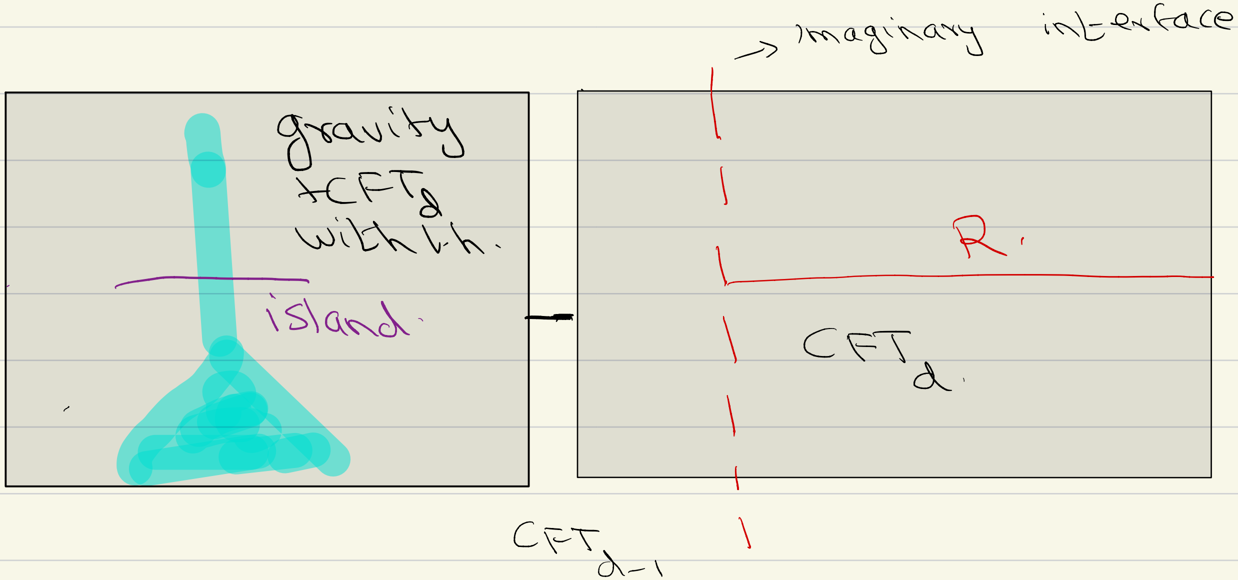
This **also leads to puzzles**, which we will resolve.

As in AdS/CFT we want to "geometrize a nongravitational question."

The island proposal provides the following answer.



Consider a cauchy slice that runs through the entire geometry



Consider all possible "islands" in the region with gravity.

$$S(R) = \min\left[\text{ext}\left[\frac{A(\partial\,\text{island})}{4G} + S_{\text{semi-cl}}(R \cup \text{island})\right]\right]$$

Meaning of $S_{\text{semi-cl}}$

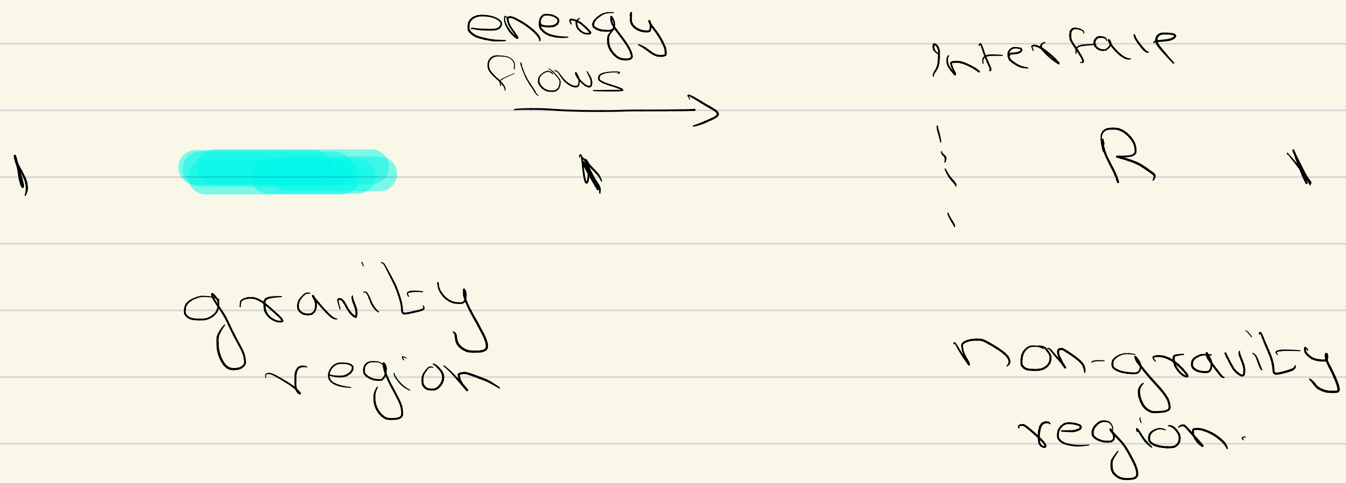
Here $S_{\text{semi-cl}}$ is defined as the entropy of the region R computed using the "rules of QFT in curved spacetime."

We start by describing the broad story of the Page curve.

We will then turn to a specific puzzle and its resolution.

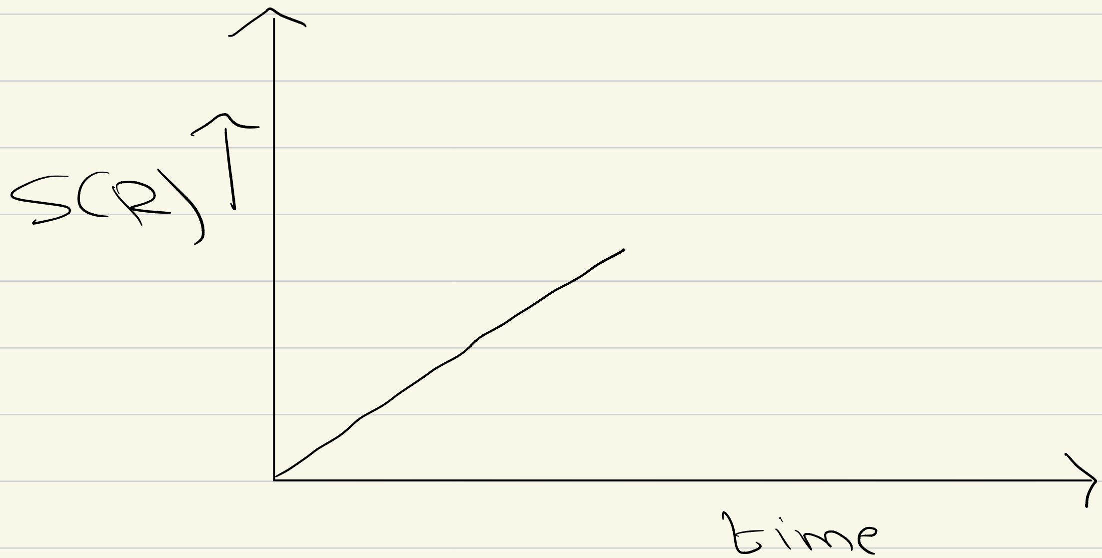
Broad idea:

Consider what happens at early times. We will draw "snapshots" in time



Initially we expect $S(R) = S_{semi-cl}(R)$

So we expect it to rise in time



Eventually S(R) can become quite large.

We expect from $S_{\text{semi-cl}}$ that

1) As time increases, it will continue to increase monotonically

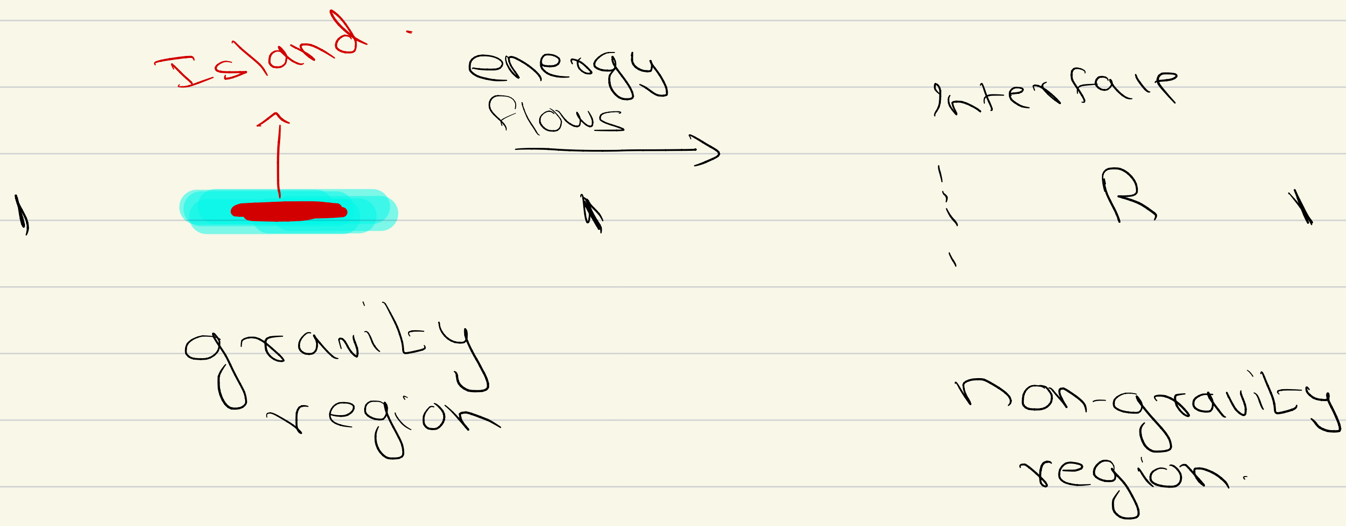
2) Nevertheless at any given time, $S_{\text{semi-cl}}$ for the

full slice

is 0.

It is not entirely clear how this $S_{\text{semi-cl}}$ should be defined in general (although for specific examples, it is understood).

But assuming properties 1 & 2, we find something interesting



Consider an "island surface" in the bulk.

This island leads to a

$$\frac{A(\partial(\text{island}))}{4G}$$

term.

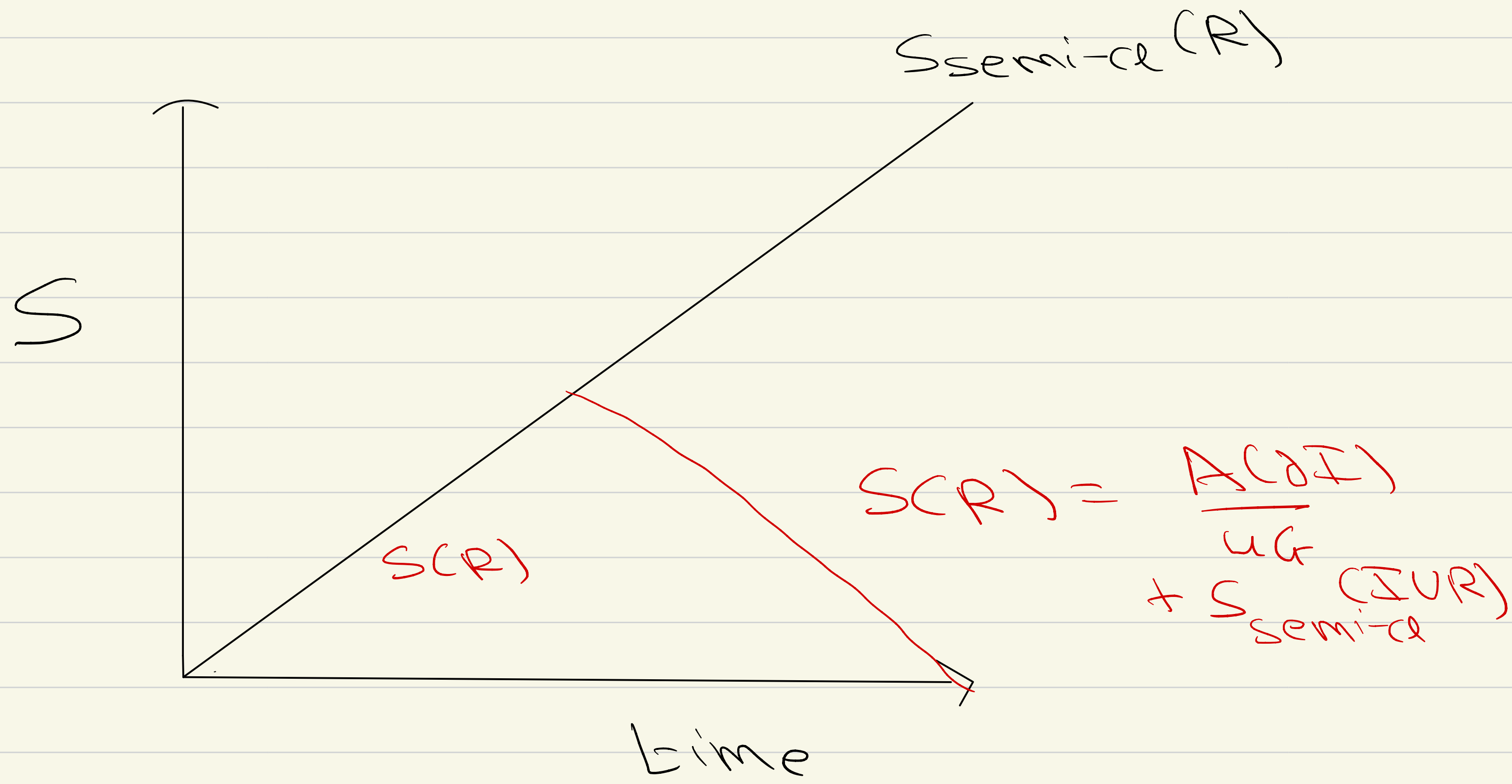
But its inclusion may also lead to a small value for $S(R \cup \text{island})$ due to property 2.

At late times, the island actually wins.

So even though $S_{semi-cl}(R)$ keeps growing, we expect that $S(R)$ starts decreasing at late times.

This is because the area of the horizon shrinks and so area of the island shrinks and $S_{semi-cl}(R)$ gets purified by a region with smaller and smaller area.

So



We emphasize this is **not** the entropy of radiation in a world with gravity everywhere but the entropy of a nongravitational region computed using holography.