Quantum Aspects of Black Holes and the Information Paradox.

Lecture 1 - 17 August 2016 - General overview

This lecture provides a broad overview of the course introducing the ideas that the course will cover

The central object in this course is a black hole. The simplest example is the Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{1-2m/r} + r^2 d\Omega^2$$

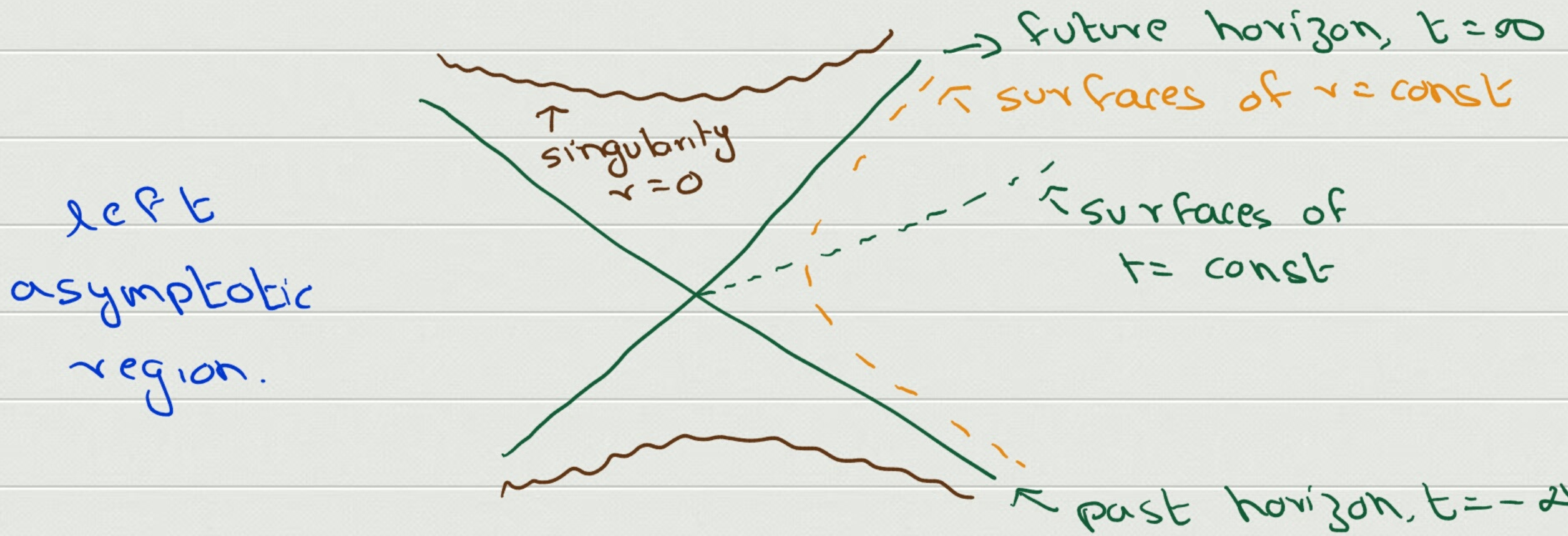
where
$$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$$
is the 2-sphere metric.

We will consider many properties of the black hole in this course.

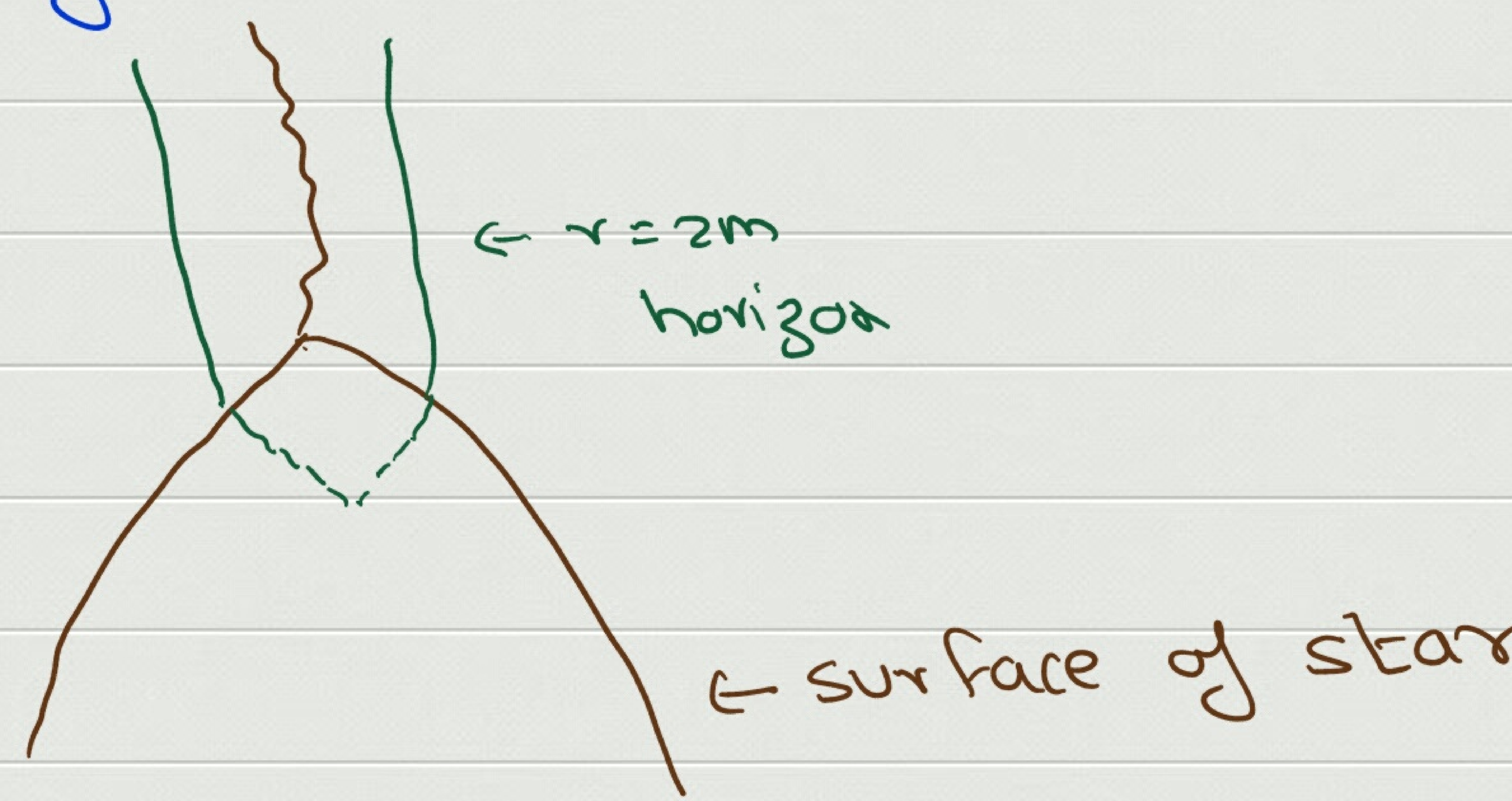
We are interested in its causal properties. As you know, the surface at $r = 2m$ has $g_{tt} = 0$; this is called a horizon.

But this region is not locally distinguished for a freely falling observer. In fact a freely falling observer can fall through the horizon in finite proper time.

This motivates the continuation of metric beyond the horizon and leads to the Kruskal diagram



We will study this diagram in detail since it is important.
But this cannot represent the diagram of a black hole formed from collapse. An example of the collapsing black hole is given by the Oppenheimer-Snyder solution, which we will examine



The reason the Schwarzschild solution is important is because of the no-hair theorem that we will review.

This states that, however the black hole was formed, it settles down to a Kerr-Newman black hole at late times (a generalization of the Schwarzschild solution with charge and angular momentum)

Even after the black hole has settled down, we can still perturb it by throwing massive objects into the black hole.

For such processes we find that

0) κ is constant throughout the horizon for a stationary b.h.

1) $dM = \frac{\kappa}{8\pi} dA + \Omega_H dJ$

2) $\delta A \geq 0$

These are analogous to the laws of thermodynamics

0) T is constant throughout a body in thermal equilibrium

1) $dU = TdS$ + work-terms

2) $\delta S \geq 0$

We will review these laws of black hole thermodynamics

Then, it was found that these were not just analogies. In fact black holes radiate like black bodies with a temperature

$$T = \frac{\kappa}{2\pi}$$

This is called Hawking radiation, and we will derive this formula.

For the Schwarzschild black hole

But, if so, then the black hole must lose mass through the Stefan-Boltzmann law

$$\frac{dM}{dt} = -\sigma T^4 A$$

For a Schwarzschild b.h., we have $T = \frac{1}{8\pi M}$ and $A = 16\pi M^2$.

So we get

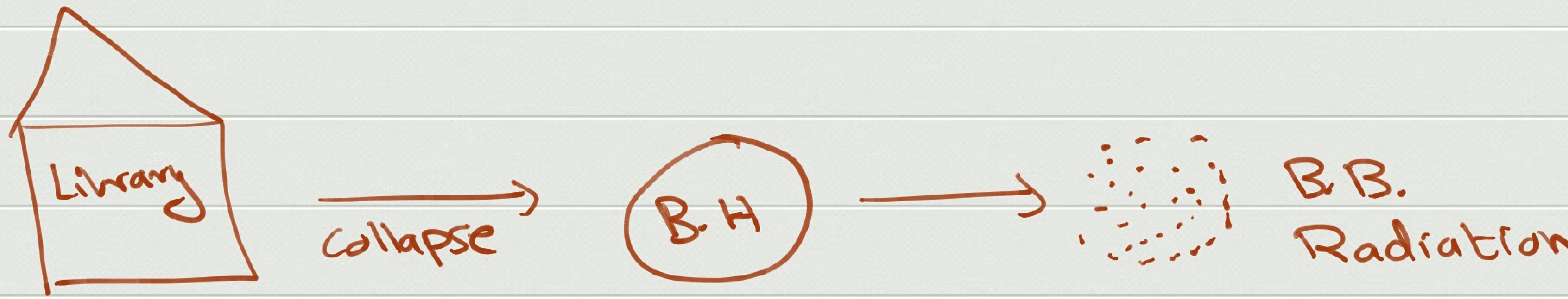
$$\frac{dM}{dt} \propto \frac{-1}{M^2}$$

$$\Rightarrow \quad t_{life} \propto M^3 \; !$$

So a black hole completely evaporates off in a time $\propto M^3$.

This leads us to the most naive version of the information paradox.
"If B.H. radiation is black body, how is evaporation consistent with unitarity?"

We seem to have the following process:



This suggests a loss of information. Multiple initial states seem to lead to the same final state. Recall, a very fundamental property of both classical & quantum mechanics is that evolution is reversible.

The Schrodinger equation also has the property that

$$i \frac{\partial |\psi\rangle}{\partial t} = H|\psi\rangle.$$

If this picture was really correct, then we would have a violation of this basic principle.

If black holes had hair, we could resolve this by appealing to the fact that Hawking radiation depends on these other parameters, and therefore can preserve information.

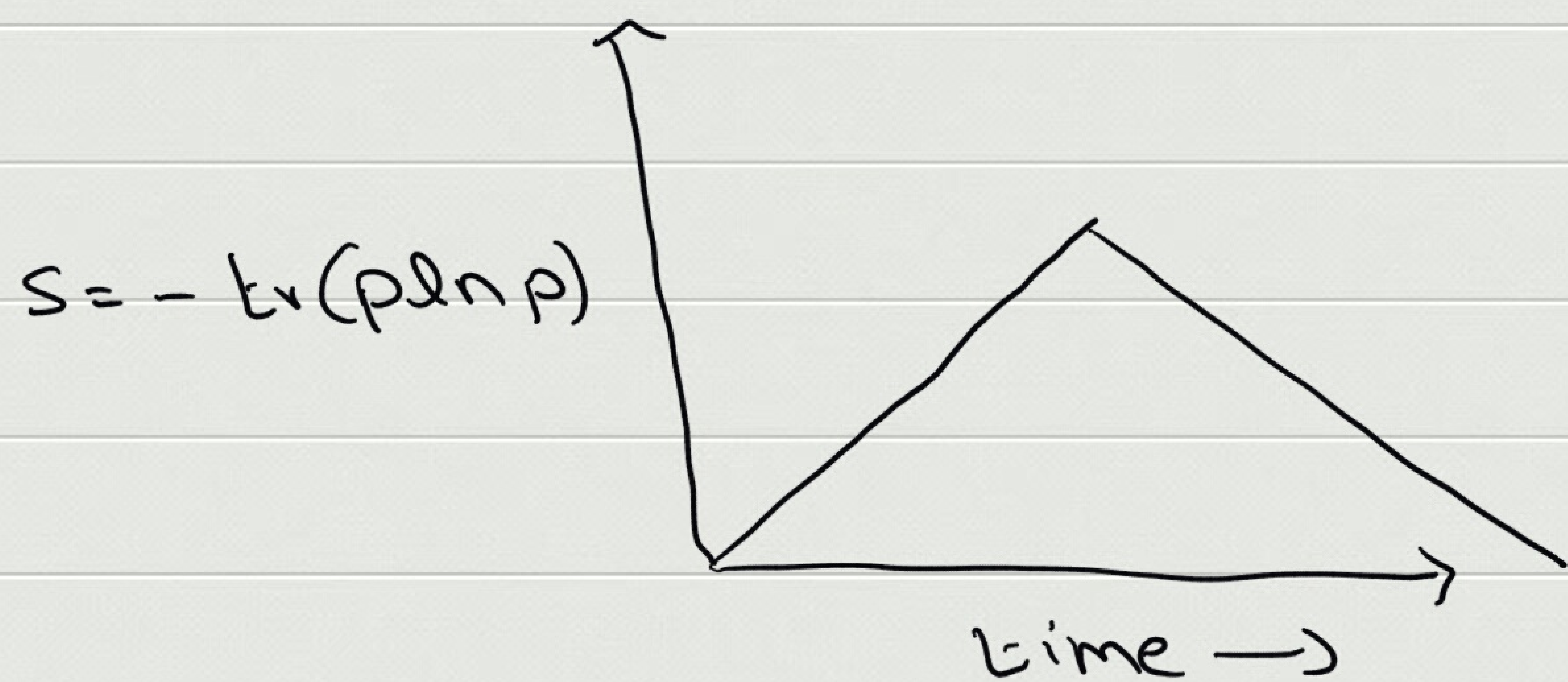
This formulation is the old information paradox:

The resolution to this paradox is as follows. Hawking's calculation tells us that some properties of the B.H. radiation are thermal. It does not tell us the radiation is exactly thermal.
Because the final phase space is so large, information can be stored in tiny correlations between different emitted quanta. Hawking's calculation is not precise enough to lead to a paradox.

We will review this resolution by examining how thermal density matrices and pure states can look almost identical for several types of observations.

But black hole evaporation is more interesting. We can see this using both older and newer arguments-

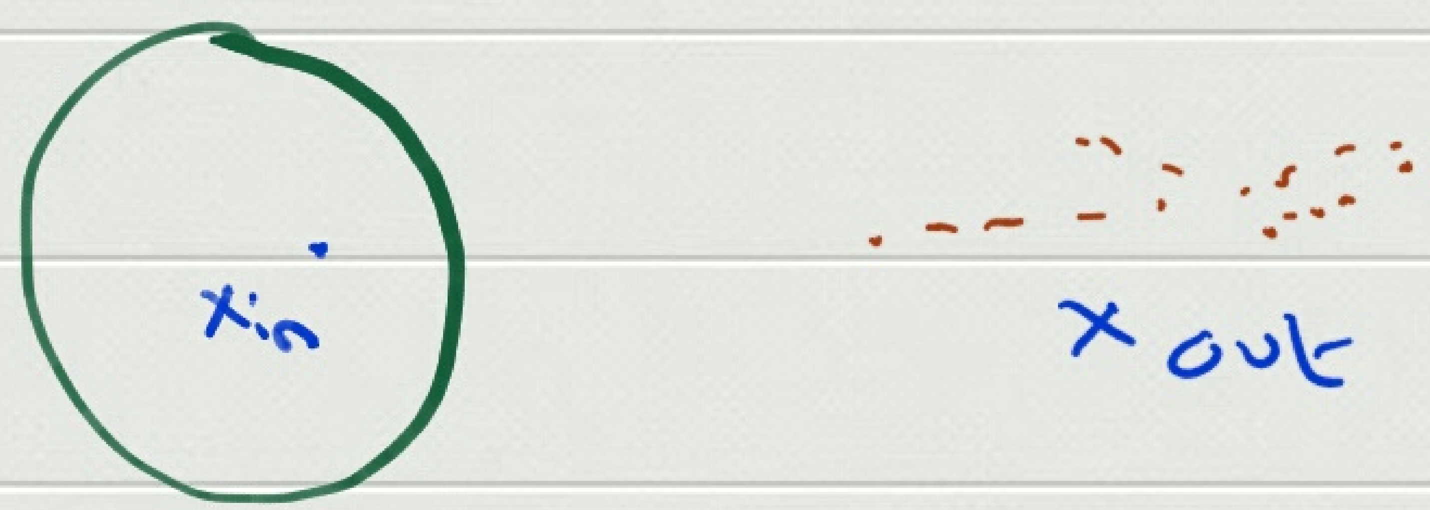
Page pointed out that, for a generic state, the von Neumann entropy of the radiation must look like



We will review the Page curve.

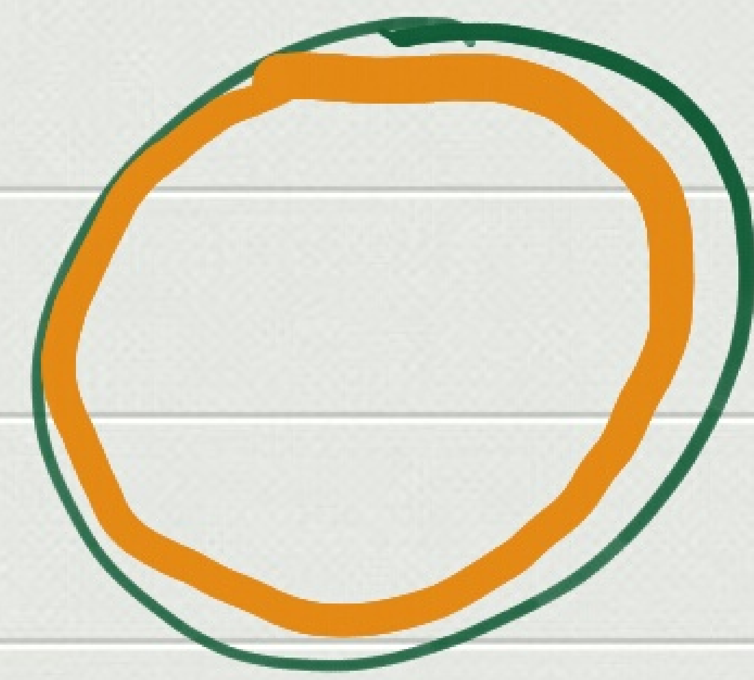
Using the nice slice and the strong subadditivity argument, it turns out that the Page curve leads to the notion of complementarity, which we will review

Since locality is not exact in quantum gravity, this is the idea that the dof inside the B.H. are scrambled versions of the dof outside the B.H.



the operator at x_{in} can be written as a scrambled version of the dof outside!!

The info paradox has also recently led to the notion of firewalls, which we will review



If true, this would be a dramatic violation of EFT

We expect Q.G. effects to be important when curvature is large, not near horizons of big black holes.

[Explain using effective action]

$$S = \frac{1}{16\pi G}\int\left(\sqrt{-g}\,R + \sqrt{-g}\,\frac{R^2}{M_{Pl}^2} + \dots\right)$$

We will also review some potential resolutions and alternatives to firewalls.